High-Level Perception as Focused Belief Revision

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Abstract. We present a framework for incorporating perception-induced beliefs into the knowledge base of a rational agent. Normally, the agent accepts the propositional content of perception and other propositions that follow from it. Given the fallibility of perception, this may result in contradictory beliefs. Hence, we model high-level perception as belief revision. We overcome difficulties imposed by the highly idealistic classical belief revision in two ways. First, we adopt a belief revision operator based on relevance logic, thus limiting the derived beliefs to those that relevantly follow from the new percept. Second, we focus belief revision on only a subset of the agent's set of beliefs—those that we take to be within the agent's current focus of attention.

1 INTRODUCTION

Evidently, perception involves some element of reflection on what is directly sensed. Neither no reflection nor unbounded reflection are appropriate. Let us refer to this kind of perception-induced reasoning, or reflection, as "high-level perception". We assume a first-order language $\mathcal{L}$, with a rich ontology including individuals, time points, acts, and states; states may be thought of as propositional fluents of the situation calculus. A sentence of the form $Holds(s, t)$ means that state s holds at time t. A functional term of the form $Prog(a)$ denotes the state that holds whenever act a is in progress. For every perceptual modality m of the agent, we shall have a predicate symbol P_m, where a sentence $P_m(s, t)$ states that the agent has a perceptual experience of state s at time t. Perception starts by an attempt to add a new belief of the form $P_m(s, ^* \mathsf{NOW})$ (now denotes the current time) to the agent's belief store.

But, as defined above, high-level perception is not the mere addition of such a belief; normally, the agent will also come to believe that s and other states (that follow from it) hold. But this might result in the agent's holding contradictory beliefs. Hence, we model high-level perception as belief revision. Adopting a classical AGM-style belief revision operator satisfying deductive closure is problematic [1], since it implies that, as a result of perception, the agent will come to believe everything that follows from its new set of beliefs. We overcome this difficulty in two ways. First, we adopt a belief revision operator based on relevance logic [2], thus limiting the derived beliefs to those that relevantly follow from the new percept. Second, we focus belief revision on only a subset of the agent's set of beliefs—those that we take to be within the agent's current focus of attention.

Work on knowledge representation aspects of perception presents multi-modal logics of the interactions between perception and belief [5, 4, 3, 7]. All these systems, however, have nothing to say about the issue of high-level perception as we described it above and about the link between perception and belief revision. Our notion of focused belief revision is related, but not identical, to the *local revision* of [6].

2 FOCUSED BELIEF REVISION

We assume a proof theory based on Anderson and Belnap's system FR of relevant implication [2]. Cn_R will be henceforth used to denote relevance logic consequence.

Definition 1 *A* ***support set*** *of a sentence $\phi \in \mathcal{L}$ is a set $s \subseteq \mathcal{L}$ such that $\phi \in \mathrm{Cn}_R(s)$. s is* ***minimal*** *if, for every $s' \subset s$, $\phi \notin \mathrm{Cn}_R(s')$.*

Definition 2 *A* ***belief state*** *$\mathbb{S}$ is a quadruple $\langle \mathcal{K}, \mathcal{B}, \sigma, \preccurlyeq \rangle$, where:*

1. *$\mathcal{K} \subseteq \mathcal{L}$ is a* ***belief set****.*
2. *$\mathcal{B} \subseteq \mathcal{K}$, with $\mathcal{K} \subseteq \mathrm{Cn}_R(\mathcal{B})$, is a finite* ***belief base****. If $\phi \in \mathcal{B}$, then ϕ is a* ***base belief****.*
3. *$\sigma : \mathcal{L} \longrightarrow 2^{2^{\mathcal{B}}}$ is a* ***support function****, where each $s \in \sigma(\phi)$ is a minimal support set of ϕ. Further, $\sigma(\phi) \neq \varnothing$ if and only if $\phi \in \mathcal{K}$. In particular, if $\phi \in \mathcal{B}$, then $\{\phi\} \in \sigma(\phi)$.*
4. *$\preccurlyeq \subseteq \mathcal{B} \times \mathcal{B}$ is a total pre-order on base beliefs.*

For brevity, where $\phi \in \mathcal{L}$ and $A \subseteq \mathcal{L}$, let $\mathrm{Cn}_R(A, \phi) = \{\psi \mid \phi \Rightarrow \psi \in \mathrm{Cn}_R(A)\}$. In what follows, $\mathbb{S} = \langle \mathcal{K}, \mathcal{B}, \sigma, \preccurlyeq \rangle$ is a belief state, $\mathcal{F} \subseteq \mathcal{K}$, and $\phi \in \mathcal{L}$.

Definition 3 *A* ***focused expansion*** *with* ***focus set*** *$\mathcal{F}$ of $\mathbb{S}$ with ϕ is a belief state $\mathbb{S} +^{\mathcal{F}} \phi = \langle \mathcal{K}_{+^{\mathcal{F}}\phi}, \mathcal{B}_{+^{\mathcal{F}}\phi}, \sigma_{+^{\mathcal{F}}\phi}, \preccurlyeq_{+^{\mathcal{F}}\phi} \rangle$, satisfying the following properties.*

(A^+1) *Success: $\mathcal{B}_{+^{\mathcal{F}}\phi} = \mathcal{B} \cup \{\phi\}$.*
(A^+2) *Relevant inclusion: $\mathcal{K}_{+^{\mathcal{F}}\phi} = \mathcal{K} \cup \mathrm{Cn}_R(\mathcal{F}, \phi)$.*
(A^+3) *Relevant Support: For every $\psi \in \mathcal{L}$,*

1. *$\sigma(\psi) \subseteq \sigma_{+^{\mathcal{F}}\phi}(\psi)$;*
2. *if $\psi \in \mathrm{Cn}_R(\mathcal{F}, \phi)$, then there is s such that, for every $s' \in \sigma_{+^{\mathcal{F}}\phi}(\phi)$, $s \cup s' \in \sigma_{+^{\mathcal{F}}\phi}(\psi)$; and*
3. *for every $s \in \sigma_{+^{\mathcal{F}}\phi}(\psi) \setminus \sigma(\psi)$, there is an s'' such that $s \in \{s'' \cup s' | s' \in \sigma_{+^{\mathcal{F}}\phi}(\phi)\} \subseteq \sigma_{+^{\mathcal{F}}\phi}(\psi)$.*

(A^+4) *Order preservation: $\preccurlyeq_{+^{\mathcal{F}}\phi}$ is a total pre-order on $\mathcal{B}_{+^{\mathcal{F}}\phi}$ such that, for every $\psi, \xi \in \mathcal{B}$, $\psi \preccurlyeq_{+^{\mathcal{F}}\phi} \xi$ if and only if $\psi \preccurlyeq \xi$.*

The belief state resulting from focused expansion by ϕ will include ϕ and anything that follows from it, given the focus set $\mathcal{F}$. That all newly derived sentences indeed follow from ϕ is guaranteed by (A^+2). In addition, old sentences may acquire new support only as a result of discovered derivations from ϕ ((A^+3)).

Definition 4 *A* ***focused revision*** *with focus set $\mathcal{F}$ of $\mathbb{S}$ with ϕ is a belief state $\mathbb{S} \dotplus^{\mathcal{F}} \phi = \langle \mathcal{K}_{\dotplus^{\mathcal{F}}\phi}, \mathcal{B}_{\dotplus^{\mathcal{F}}\phi}, \sigma_{\dotplus^{\mathcal{F}}\phi}, \preccurlyeq_{\dotplus^{\mathcal{F}}\phi} \rangle$, satisfying the following properties.*

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$(A^{\dot{+}}1)$ *Base inclusion:* $\mathcal{B}_{\dot{+}^{\mathcal{F}}\phi} \subseteq \mathcal{B}_{+^{\mathcal{F}}\phi}$.
$(A^{\dot{+}}2)$ *Inclusion:* $\mathcal{K}_{\dot{+}^{\mathcal{F}}\phi} \subseteq \mathcal{K}_{+^{\mathcal{F}}\phi}$.
$(A^{\dot{+}}3)$ *Lumping:* $\psi \in \mathcal{K}_{+^{\mathcal{F}}\phi} \backslash \mathcal{K}_{\dot{+}^{\mathcal{F}}\phi}$ *if and only if, for every* $s \in \sigma_{+^{\mathcal{F}}\phi}(\psi)$, $s \not\subseteq \mathcal{B}_{\dot{+}^{\mathcal{F}}\phi}$.
$(A^{\dot{+}}4)$ *Preferential core-retainment:* $\psi \in \mathcal{B}_{+^{\mathcal{F}}\phi} \backslash \mathcal{B}_{\dot{+}^{\mathcal{F}}\phi}$ *if and only if there is* $\chi \in \mathcal{L}$ *such that* $(\chi \wedge \neg\chi) \in \mathrm{Cn}_R(\mathcal{F}, \phi)$ *and there is* $s \in \sigma_{+^{\mathcal{F}}\phi}(\chi \wedge \neg\chi)$ *such that* ψ *is a minimal element of* s *with respect to* $\preccurlyeq_{+^{\mathcal{F}}}$.
$(A^{\dot{+}}5)$ *Support update: For every* $\psi \in \mathcal{L}$, $\sigma_{\dot{+}^{\mathcal{F}}\phi}(\psi) = \sigma_{+^{\mathcal{F}}\phi}(\psi) \cap 2^{\mathcal{B}_{\dot{+}^{\mathcal{F}}\phi}}$
$(A^{\dot{+}}6)$ *Order preservation:* $\preccurlyeq_{\dot{+}^{\mathcal{F}}\phi}$ *is the restriction of* $\preccurlyeq_{+^{\mathcal{F}}\phi}$ *to* $\mathcal{B}_{\dot{+}^{\mathcal{F}}\phi}$.

Thus, focused revision is focused expansion followed by some kind of consolidation. Consolidation is implemented by removing least-preferred beliefs (as per $\preccurlyeq$) from each support set of a contradiction ($\chi \wedge \neg\chi$) in the inconsistent belief state resulting from expansion by ϕ.

3 THE FOCUS SET

To model high-level perception by focused belief revision, we need to consider interpretations of the focus set $\mathcal{F}$ that are suitable for perception. We believe that the selection of a suitable focus set should be based on (at least) three factors: (i) what is vital for the agent, (ii) what is relevant for the agent, and (iii) how much resources are available for perception-induced reasoning.

For every agent, there are certain things that it cannot afford to not notice. For example, an agent might believe that, whenever there is fire, it should leave the building. A focus set of such an agent must include beliefs that allow it to conclude the imminence of fire from the perception of signs of fire.

We take beliefs relevant to the agent to be those relevant to what the agent believes itself to be doing. In this paper, we adopt a simple syntactic indicator of relevance that we call n^{th}*-degree term sharing*. As it turns out, the degree of term sharing provides a way to tune the construction of the focus set to the amount of resources the agent can spend in the process.

For any $\mathbb{S} = \langle \mathcal{K}, \mathcal{B}, \sigma, \preccurlyeq \rangle$, $\alpha(\mathbb{S})$ (or α when $\mathbb{S}$ is obvious) is the set of all sentences in $\mathcal{K}$ of the form $Holds(Prog(a), ^{*}\mathsf{NOW})$. A function $\gamma : 2^{\mathcal{L}} \times \mathcal{L} \longrightarrow 2^{\mathcal{L}}$ is a **relevance filtering function** if $\gamma(A, \phi) \subseteq A$. If $\phi \in \mathcal{L}$, $\tau(\phi)$ is the set of all closed terms occurring in ϕ and $TS(\phi) = \{\psi | \psi \in \mathcal{K} \cup \{\phi\} \text{ and } \tau(\phi) \cap \tau(\psi) \neq \varnothing\}$.

Definition 5 *Let* $n \in \mathbb{N}$ *and let* γ *be a relevance filtering function. An* n^{th}***-degree term sharing function with filter*** γ *is a function* $\mathfrak{t}^n_\gamma : \mathcal{L} \longrightarrow 2^{\mathcal{L}}$ *defined as follows:*

$$\mathfrak{t}^n_\gamma(\phi) = \begin{cases} \{\phi\} & \text{if } n = 0 \\ \gamma(TS(\phi), \phi) & \text{if } n = 1 \\ \{\psi | \text{ for some } \xi \in \mathfrak{t}^{n-1}_\gamma(\phi), \psi \in \mathfrak{t}^1_\gamma(\xi)\} & \text{otherwise} \end{cases}$$

$\mathfrak{t}^1_\gamma(\phi)$ is the result of filtering the set of sentences that share at least one term with ϕ. The filtering function, which is largely agent-dependent, is used to account for the fact that term sharing is not sufficient for relevance.

In what follows, $\mathbb{S} = \langle \mathcal{K}, \mathcal{B}, \sigma, \preccurlyeq \rangle$ and $p = P_m(s, ^{*}\mathsf{NOW}) \in \mathcal{L}$.

Definition 6 *A* ***focus structure*** $\mathbb{F}_{\mathbb{S},p}$ *is a quadruple* $\langle \mathcal{V}, \Gamma, \Delta, \rho \rangle$, *where*

- $\mathcal{V} \subseteq \mathcal{K}$ *is a set of* ***vital*** *beliefs,*
- $\Gamma : \alpha \cup \{p\} \longrightarrow [2^{\mathcal{L}} \times \mathcal{L} \longrightarrow 2^{\mathcal{L}}]$,
- $\Delta : \alpha \cup \{p\} \longrightarrow \mathbb{N}$, *and*
- $\rho : 2^{\mathcal{L}} \times 2^{2^{\mathcal{L}}} \longrightarrow 2^{\mathcal{L}}$ *is a* ***relevance choice function****, where*

$$\rho(\mathfrak{t}^{\Delta(p)}_{\Gamma(p)}(p), \{\mathfrak{t}^{\Delta(a)}_{\Gamma(a)}(a)\}_{a \in \alpha}) \subseteq \bigcup_{\phi \in \alpha \cup \{p\}} \mathfrak{t}^{\Delta(\phi)}_{\Gamma(\phi)}(\phi)$$

The above notion of focus structure is an attempt to pinpoint the factors contributing to the construction of focus sets. Nonetheless, the definition is flexible enough to accommodate agent-specific considerations regarding the exact components of the focus structure.

Definition 7 *Let* $\mathbb{F}_{\mathbb{S},p} = \langle \mathcal{V}, \Gamma, \Delta, \rho \rangle$ *be a focus structure. The* ***high-level perception*** *of* s *in* $\mathbb{S}$ *with focus structure* $\mathbb{F}_{\mathbb{S},p}$ *is the focused belief revision,* $\mathbb{S} \dot{+}^{\mathcal{F}} p$, *of* $\mathbb{S}$ *with* p *where*

$$\mathcal{F} = \mathcal{V} \cup \Pi \cup \rho(\mathfrak{t}^{\Delta(p)}_{\Gamma(p)}(p), \{\mathfrak{t}^{\Delta(a)}_{\Gamma(a)}(a)\}_{a \in \alpha})$$

and p *is a maximal element of* $\mathcal{B}_{+^{\mathcal{F}}p}$ *with respect to* $\preccurlyeq_{+^{\mathcal{F}}p}$.

The set Π appearing in the definition of $\mathcal{F}$ above is a perception theory allowing us to derive $Holds(s, t)$ from $P_m(s, t)$ under normal circumstances. The requirement that p be a maximally preferred belief reflects the idea (often discussed in the philosophical literature) that having a present perceptual experience of some state s is indefeasible.

4 CONCLUSION

We have presented a framework for high-level perception as focused belief revision. This simultaneously addresses two issues. On one hand, the defeasibility of perception-induced beliefs is accounted for through the underlying reason maintenance system. On the other hand, bounded reflection on the contents of perception is implemented by two aspects of our system. First, the use of relevance logic guarantees that all perception-induced beliefs follow from the perception belief itself. Second, the definition of the focus set presented limits reasoning only to what is relevant and vital for the agent, while taking issues of resource boundedness into account in a fairly general way.

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