Data Integration: Logic Query Languages

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A logic language

- Datalog programs consist of logical facts and rules
- Datalog is a subset of Prolog (no data structures)

Basic concepts

- term: constant, variable
- predicate (relation)
- atom
- clause, rule, fact
- groundness: no variables
- substitution: mapping from variables to terms
- ground substitution: mapping from variables to constants

Atom

- syntax: $P(T_1, \ldots, T_n)$
- semantics: predicate P is true of terms T_1 and ... and T_n
- variables present: truth under a ground substitution

Implication (clause)

- syntax: $A_0 \ :- \ A_1, \ldots, A_k$
- semantics: atom A_0 is true if atoms A_1 and $\cdots$ and A_k are true
- all the variables universally quantified:
  - implication true under all ground substitutions
- all the variables in the head occur also in the body
- some predicates in the body may be built-in: $>, \geq, \ldots$

Kinds of clauses

- $k = 0$: fact
- $k > 0$: rule consisting of head A_0 and body $A_1, \ldots, A_k$

Datalog program P

- $EDB(P)$: a set of true ground facts encoding a database instance
- $IDB(P)$: a set of rules encoding a query, with a special predicate `query` to return the result

Predicate dependence

- direct: the predicate in the head depends on each predicate in the body
- indirect: through multiple rules
- recursion: predicate depends on itself

Logic query languages

- conjunctive queries: single rule, no recursion
- unions of conjunctive queries: multiple rules, no recursion
- Datalog: multiple rules, recursion

Example logic program

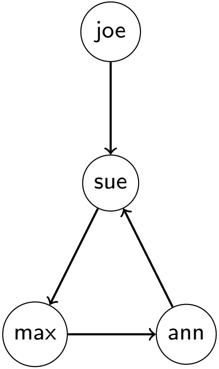
Friends

```
%% Facts
friend(joe,sue).
friend(ann,sue).
friend(sue,max).
friend(max,ann).

%% Rules
fof(X,Y) :- friend(X,Y).
fof(X,Z) :- friend(X,Y), fof(Y,Z).

%% Query 1
query(X) :- fof(X,ann).

%% Query 2
query(X) :- fof(X,Y), fof(Y,X).
```



Logical semantics

Deriving facts of *IDB*(*P*) predicates bottom-up

1. *EDB*(*P*) facts are true
2. single step: for all the ground substitutions that map the body of a rule in *IDB*(*P*) to true facts, make the (substituted) head true
3. repeat until no new true facts are derived

Derivation properties

- the derivation terminates: why?
- soundness: the derived true facts are logical consequences of *P*
- completeness: all the logical consequences of *P* are derived

Derived facts: all `friend` facts and

```
%% Direct friends
fof(joe,sue).
fof(ann,sue).
fof(sue,max).
fof(max,ann).

%% Second-level friends
fof(joe,max).
fof(sue,ann).
fof(ann,max).
fof(max,sue).

%% Third-level friends
fof(joe,ann).
fof(sue,sue).
for(ann,ann).
for(max,max).
```

Open vs. Closed World Assumption

Closed World Assumption (CWA)

What is not implied by a logic program is false.

Open World Assumption (OWA)

What is not implied by a logic program is unknown.

Scope

- traditional database applications: CWA
- information integration: OWA or CWA

Can negation be allowed inside Datalog rules?

Datalog^{not}

Syntax

Rules with negative atoms in the body:

$$A_0 :- A_1, \ldots, A_k, \textit{not}\ B_1, \ldots, \textit{not}\ B_m.$$

Example

```
asymmetric(X,Y):- fof(X,Y), not fof(Y,X).
```

Semantics

Cannot be adequately given in terms of implication.

Coding relational algebra

Coding operators

- selection, Cartesian product: single clause
- projection: single clause with projected out attributes only in the body
- union: multiple clauses
- set difference: negation

But what about recursion?

Dependency graph $pdg(P)$

- vertices: predicates of a Datalog^{not} program P
- edges:
  - a positive edge (p, q) if there is a clause in P in which q appears in a positive atom in the body and p appears in the head
  - a negative edge (p, q) if there is a clause in P in which q appears in a negative atom in the body and p appears in the head

Stratified P

No cycle in $pdg(P)$ contains a negative edge.

Stratification

Mapping s from the set of predicates in P to nonnegative integers such that:

1. if a positive edge (p, q) is in $pdg(P)$, then $s(p) \geq s(q)$
2. if a negative edge (p, q) is in $pdg(P)$, then $s(p) > s(q)$

There is a polynomial-time algorithm to determine whether a program is stratified, and if it is, to find a stratification for it.

Stratified Datalog^{not}: query evaluation

Bottom-up evaluation

1. compute a stratification of a program P
2. partition P into $P_1, \ldots, P_n$ such that
   - each P_i consisting of all and only rules whose head belongs to a single stratum
   - P_1 is the lowest stratum
3. evaluate bottom-up $P_1, \ldots, P_n$ (in that order).

Result

- does not depend on the stratification
- can be semantically characterized in various ways
- is used to compute query results

Coding universal quantification through double negation.

Example

```
everybodysFriend(X):- person(X), not isNotFriend(X).
isNotFriend(X):- person(X), person(Y), not friend(Y,X).
```

Query result

The query result $Q(D)$ is defined for every input database D.

Query containment

$$Q_1 \sqsubseteq Q_2 \text{ if and only if } Q_1(D) \subseteq Q_2(D) \text{ for every input database } D.$$

Query equivalence

$$Q_1 \equiv Q_2 \text{ if and only if } Q_1 \sqsubseteq Q_2 \text{ and } Q_2 \sqsubseteq Q_1.$$

Query language containment

$L_1 \subseteq L_2$ if for every query $Q_1 \in L_1$, there is an equivalent query Q_2 in L_2.

- Q_2 may be in a different syntax than Q_1

Comparing query languages

Expressiveness

- Datalog $\subseteq$ Stratified Datalog^{not}
- Relational Algebra $\subseteq$ Stratified Datalog^{not}
- Datalog $\not\subseteq$ Relational Algebra
  - transitive closure
- Relational Algebra $\not\subseteq$ Datalog
  - set difference

How to prove expressiveness results?

- considering the syntax not enough
- semantic propreties

Monotonicity

A query language L is monotonic if for every query $Q \in L$, adding facts to the database cannot remove any tuples from the result of Q. Formally: for all databases D_1 and D_2

$$D_1 \subseteq D_2 \text{ implies } Q(D_1) \subseteq Q(D_2).$$

Query languages

- monotonic: Datalog
- nonmonotonic: Datalog^{not}, relational algebra, SQL