Problem 1

Assume α is a selection condition on a relation schema R. A new relational algebra operator $A_{\alpha}(R)$ is defined in the following way:

$$A_{\alpha}(r) = \begin{cases} r & \text{if } \sigma_{\alpha}(r) = \emptyset \\ \sigma_{\alpha}(r) & \text{if } \sigma_{\alpha}(r) \neq \emptyset \end{cases}$$

for any instance r of R.

Express $A_{\alpha}(R)$ using the basic operators of the relational algebra.

Problem 2

You are given a relation with N columns of the same type. Write an SQL query that returns the tuples having the *maximum number of repetitions* of the same value. The query should have size polynomial in N.

Problem 3

Assume a tree is represented as a set of facts of the form parent(x, y) where x is a parent of y in the tree. Write a Datalog program that checks whether two given tree nodes are at the same distance from the root. Test this program using XSB Prolog.

You may use the built-in XSB Prolog predicate $x \ge y$ meaning that x is different from y. You may not use negation or aggregation. Turn auto-tabling on by including the directive

```
:- auto_table.
```

in your source file. The source file should have the extension .P.