Foundations of Preference Queries

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Plan of the course

1. Preference relations
2. Preference queries
3. Preference management
4. Advanced topics

Part I

Preference relations

Outline of Part I

1. Preference relations
   - Preference
   - Equivalence
   - Preference specification
   - Combining preferences
   - Skylines

Preference relations

Universe of objects

- constants: uninterpreted, numbers,...
- individuals (entities)
- tuples
- sets

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Preference relation $\succ$

- binary relation between objects
- $x \succ y \equiv x$ *is_better_than* $y \equiv x$ dominates y
- an abstract, uniform way of talking about desirability, worth, cost, timeliness,..., and their combinations
- preference relations used in queries

Buying a car

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Salesman: Which is more important for you: the age or the price?
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Salesman: Those are the best cars, according to your preferences, that we have in stock.

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Customer: The age, definitely.
Salesman: Those are the best cars, according to your preferences, that we have in stock.
Customer: Wait...it better be a BMW.

Applications of preferences and preference queries

1. decision making
2. e-commerce
3. digital libraries
4. personalization

Properties of preference relations

Properties of preference relations

Properties of $\succ$

- irreflexivity: $\forall x.\ x \not\succ x$
- asymmetry: $\forall x, y.\ x \succ y \Rightarrow y \not\succ x$
- transitivity: $\forall x, y, z.\ (x \succ y \wedge y \succ z) \Rightarrow x \succ z$
- negative transitivity: $\forall x, y, z.\ (x \not\succ y \wedge y \not\succ z) \Rightarrow x \not\succ z$
- connectivity: $\forall x, y.\ x \succ y \vee y \succ x \vee x = y$

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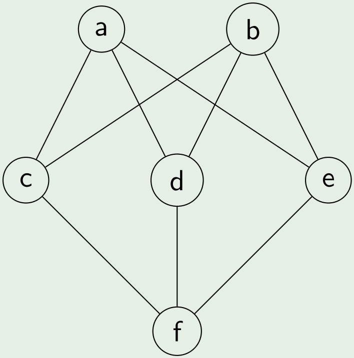
Orders

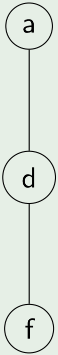
- strict partial order (SPO): irreflexive and transitive
- weak order (WO): negatively transitive SPO
- total order: connected SPO

Weak and total orders

Weak order



Total order



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- scoring functions represent weak orders

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We assume that preference relations are SPOs.

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Several notions of equivalence

- equality: $x \sim^{eq} y \;\equiv\; x = y$
- indifference: $x \sim^{i} y \;\equiv\; x \not\succ y \wedge y \not\succ x$
- restricted indifference:
  $x \sim^{r} y \equiv \forall z. \, (x \prec z \Leftrightarrow y \prec z) \wedge (z \prec y \Leftrightarrow z \prec x)$

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Properties of equivalence

- equivalence relation: reflexive, symmetric, transitive
- equality and restricted indifference (if $\succ$ is an SPO) are equivalence relations
- indifference is reflexive and symmetric; transitive for WO

Example

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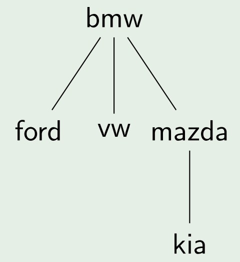

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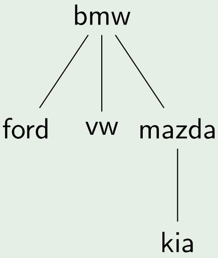

Preference:

$bmw \succ ford$, $bmw \succ vw$
$bmw \succ mazda$, $bmw \succ kia$
$mazda \succ kia$

Indifference:

$ford \sim^i vw$, $vw \sim^i ford$,
$ford \sim^i mazda$, $mazda \sim^i ford$,
$vw \sim^i mazda$, $mazda \sim^i vw$,
$ford \sim^i kia$, $kia \sim^i ford$,
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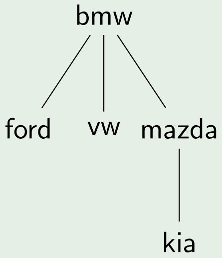

This is a strict partial order which is not a weak order.

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Not every SPO is a WO

Canonical example

$mazda \succ kia,\ mazda \sim^i vw,\ kia \sim^i vw$

Violation of negative transitivity

$mazda \not\succ vw,\ vw \not\succ kia,\ mazda \succ kia$

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- can be infinite but finitely representable
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$$(m_1, y_1, p_1) \succ_1 (m_2, y_2, p_2) \equiv y_1 > y_2 \vee (y_1 = y_2 \wedge p_1 < p_2)$$

for relation $Car(Make, Year, Price)$.

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- defined using real-valued scoring functions:

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- defined using preference constructors (Preference SQL)
- defined using real-valued scoring functions: $F(m, y, p) = \alpha \cdot y + \beta \cdot p$
  $(m_1, y_1, p_1) \succ_2 (m_2, y_2, p_2) \equiv F(m_1, y_1, p_1) > F(m_2, y_2, p_2)$

Logic formulas

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The language of logic formulas

- constants
- object (tuple) attributes
- comparison operators: $=, \neq, <, >, \ldots$
- arithmetic operators: $+, \cdot, \ldots$
- Boolean connectives: $\neg, \wedge, \vee$
- quantifiers:
    - $\forall, \exists$
    - usually can be eliminated (quantifier elimination)

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Sufficient condition for representability

- $\succ$ is a weak order
- the domain is countable or some continuity conditions are satisfied (studied in decision theory)

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Lexicographic order in $R \times R$

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7. Each such interval contains a rational number: contradiction with the countability of the set of rational numbers.

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Combining preferences

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Preference accumulation

- defining preferences over objects in terms of preferences over simpler objects
- dimensionality is increased (preferences over Cartesian product).

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Pareto composition

$$x \succ^{Par} y \equiv (x \succ_1 y \wedge y \not\succ_2 x) \vee (x \succ_2 y \wedge y \not\succ_1 x).$$

Preference composition

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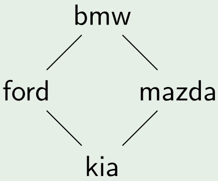
Preference relation $\succ_1$



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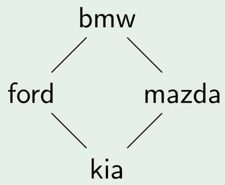
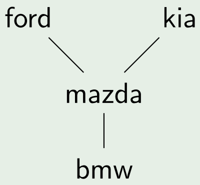


Preference relation $\succ_2$



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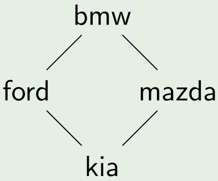

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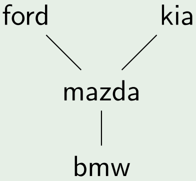

Prioritized composition

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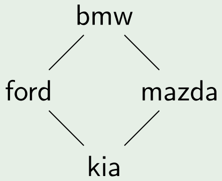

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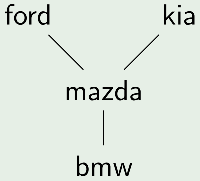
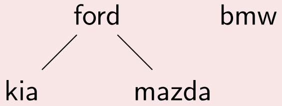


Prioritized composition

Pareto composition



Combining preferences: accumulation [Kie02]

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Prioritized accumulation: $\succ^{pr} = (\succ_1 \& \succ_2)$

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Properties

- closure
- associativity
- commutativity of Pareto accumulation

Skylines

Skylines

Skyline

Given single-attribute total preference relations $\succ_{A_1}, \ldots, \succ_{A_n}$ for a relational schema $R(A_1, \ldots, A_n)$, the skyline preference relation $\succ^{sky}$ is defined as

$$\succ^{sky} = \succ_{A_1} \otimes \succ_{A_2} \otimes \cdots \otimes \succ_{A_n} .$$

Unfolding the definition

$$(x_1, \ldots, x_n) \succ^{sky} (y_1, \ldots, y_n) \equiv \bigwedge_i x_i \succeq_{A_i} y_i \wedge \bigvee_i x_i \succ_{A_i} y_i .$$

Skyline in Euclidean space

Skyline in Euclidean space

Two-dimensional Euclidean space

$$(x_1, x_2) \succ^{sky} (y_1, y_2) \equiv x_1 \geq y_1 \wedge x_2 > y_2 \vee x_1 > y_1 \wedge x_2 \geq y_2$$

Skyline consists of $\succ_{sky}$-maximal vectors

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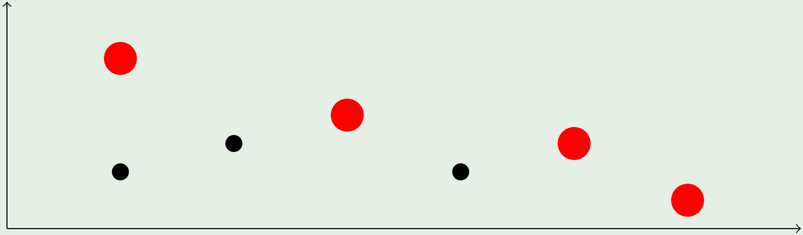

Skyline properties

Skyline properties

Invariance

A skyline preference relation is unaffacted by scaling or shifting in any dimension.

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A skyline preference relation is unaffected by scaling or shifting in any dimension.

Maxima

A skyline consists of the maxima of monotonic scoring functions.

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Invariance

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Maxima

A skyline consists of the maxima of monotonic scoring functions.

Skyline is not a weak order

$$(2,0) \not\succ_{sky} (0,2), (0,2) \not\succ_{sky} (1,0), (2,0) \succ_{sky} (1,0)$$

Skyline in SQL

Skyline in SQL

Grouping

Designating attributes not used in comparisons (DIFF).

Example

```
SELECT * FROM Car
SKYLINE Price MIN,
        Year MAX,
        Make DIFF
```

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Dynamic skylines

- dimensions defined using dimension functions $g_1, \ldots, g_n$
- variable query point.

Dynamic skylines

Dynamic skylines

Relation $Hotel(XCoord, YCoord, Price)$

- tuple $p = (p_x, p_y, p_z)$, query point (u_x, u_y)
- dimension functions based on 2D Euclidean distance:

$$g_1(p_x, p_y) = \sqrt{(p_x - u_x)^2 + (p_y - u_y)^2}$$

$$g_2(p_z) = p_z$$

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XCoord	YCoord	Price
0	5	80
2	6	100
5	3	120

Query point: (3,4).

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Common scenario

- scoring functions $f_1, \ldots, f_n$
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- linear scoring function: $\Sigma_{i=1}^{n} \alpha_i f_i$

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Numerical vs. logical combination

- logical combination cannot be defined numerically
- numerical combination cannot be defined logically (unless arithmetic operators are available)

Part II

Preference Queries

Outline of Part II

2. Preference queries
   - Retrieving non-dominated elements
   - Rewriting queries with winnow
   - Retrieving Top-K elements
   - Optimizing Top-K queries

Winnow[Cho03]

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- new relational algebra operator ω (other names: Best, BMO [Kie02])
- retrieves the non-dominated (best) elements in a database relation
- can be expressed in terms of other operators

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Definition

Given a preference relation $\succ$ and a database relation r:

$$\omega_{\succ}(r) = \{t \in r \mid \neg\exists t' \in r.\ t' \succ t\}.$$

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Skyline query

$\omega_{\succ_{sky}}(r)$ computes the set of maximal vectors in r (the skyline set).

Example of winnow

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Relation $Car(Make, Year, Price)$

Preference relation:

$$(m, y, p) \succ_1 (m', y', p') \equiv y > y' \lor (y = y' \land p < p').$$

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Computing winnow using BNL [BKS01]

Require: SPO $\succ$, database relation r

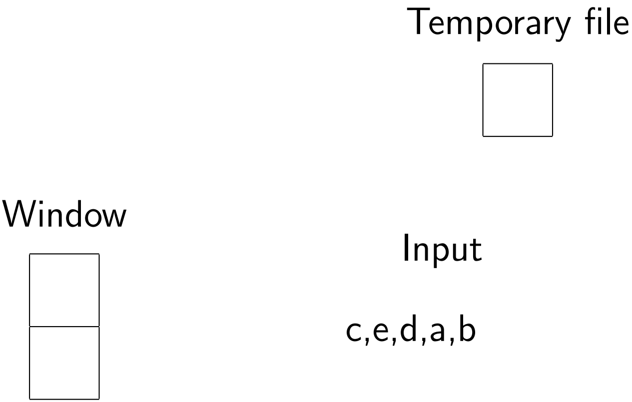
```
initialize window W and temporary file F to empty
repeat
    for every tuple t in the input do
        if t is dominated by a tuple in W then
            ignore t
        else if t dominates some tuples in W then
            eliminate them and insert t into W
        else if there is room in W then
            insert t into W
        else
            add t to F
        end if
    end for
    output tuples from W that were added when F was empty
    make F the input, clear F
until empty input
```

BNL in action

Preference relation: a $\succ$ c, a $\succ$ d, b $\succ$ e.

BNL in action

Preference relation: $a \succ c$, $a \succ d$, $b \succ e$.



BNL in action

Preference relation: a $\succ$ c, a $\succ$ d, b $\succ$ e.

Temporary file

Window

c

Input

e,d,a,b

BNL in action

Preference relation: $a \succ c$, $a \succ d$, $b \succ e$.

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Window

c

e

Input

d,a,b

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d

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Input

a,b

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d

Window

a
e

Input

b

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a
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a
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Preference relation: $a \succ c$, $a \succ d$, $b \succ e$.

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a

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Input

Computing winnow with presorting

Computing winnow with presorting

SFS: adding presorting step to BNL [CGGL03]

- topologically sort the input:
  - if x dominates y, then x precedes y in the sorted input
  - window contains only winnow points and can be output after every pass
- for skylines: sort the input using a monotonic scoring function, for example $\prod_{i=1}^{k} x_i$.

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LESS: integrating different techniques [GSG07]

- adding an elimination filter to the first external sort pass
- combining the last external sort pass with the first SFS pass
- average running time: $O(kn)$

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Preference relation: a $\succ$ c, a $\succ$ d, b $\succ$ e.

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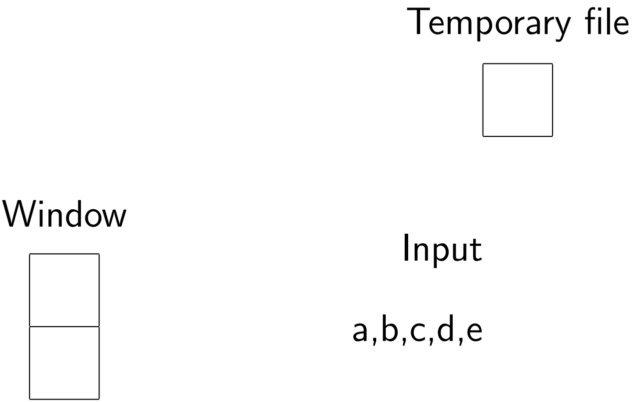

SFS in action

Preference relation: a $\succ$ c, a $\succ$ d, b $\succ$ e.

Temporary file

Window

a

Input

b,c,d,e

SFS in action

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Window

a
b

Input

c,d,e

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Temporary file

Window

a
b

Input

d,e

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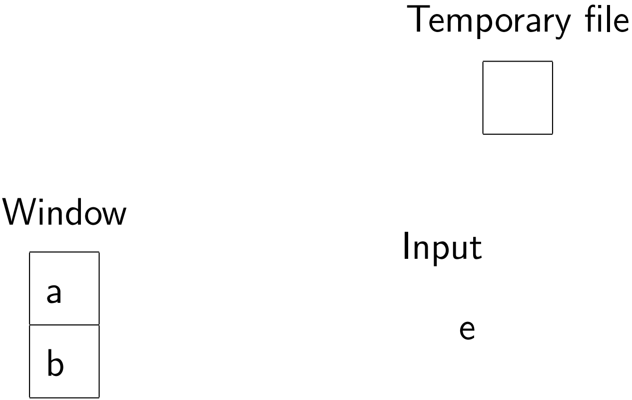

SFS in action

Preference relation: a $\succ$ c, a $\succ$ d, b $\succ$ e.

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a

b

Input

Generalizations of winnow

Generalizations of winnow

Iterating winnow

$$\omega^0_{\succ}(r) = \omega_{\succ}(r)$$

$$\omega^{n+1}_{\succ}(r) = \omega_{\succ}(r - \bigcup_{1 \leq i \leq n} \omega^i_{\succ}(r))$$

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Ranking

Rank tuples by their minimum distance from a winnow tuple:

$$\eta_\succ(r) = \{(t, i) \mid t \in \omega^i_C(r)\}.$$

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Iterating winnow

$$\omega^0_{\succ}(r) = \omega_{\succ}(r)$$
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k-band

Return the tuples dominated by at most k tuples:

$$\omega_{\succ}(r) = \{t \in r \mid \#\{t' \in r \mid t' \succ t\} \leq k\}.$$

Preference SQL

Preference SQL

The language

- basic preference constructors
- Pareto/prioritized accumulation
- new SQL clause `PREFERRING`
- groupwise preferences
- implementation: translation to SQL

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Winnow in Preference SQL

```
SELECT * FROM Car
PREFERRING HIGHEST(Year)
    CASCADE LOWEST(Price)
```

Algebraic laws [Cho03]

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Commutativity of winnow with selection

If the formula

$$\forall t_1, t_2.[\alpha(t_2) \wedge \gamma(t_1, t_2)] \Rightarrow \alpha(t_1)$$

is valid, then for every r

$$\sigma_\alpha(\omega_\gamma(r)) = \omega_\gamma(\sigma_\alpha(r)).$$

Algebraic laws [Cho03]

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is valid, then for every r

$$\sigma_\alpha(\omega_\gamma(r)) = \omega_\gamma(\sigma_\alpha(r)).$$

Under the preference relation

$$(m, y, p) \succ_{C_1} (m', y', p') \equiv y > y' \wedge p \leq p' \vee y \geq y' \wedge p < p'$$

the selection $\sigma_{Price<20K}$ commutes with ω_{C_1} but $\sigma_{Price>20K}$ does not.

Other algebraic laws

Other algebraic laws

Distributivity of winnow over Cartesian product

For every r_1 and r_2

$$\omega_C(r_1 \times r_2) = \omega_C(r_1) \times r_2$$

if C refers only to the attributes of r_1.

Commutativity of winnow

If $\forall t_1, t_2.[C_1(t_1, t_2) \Rightarrow C_2(t_1, t_2)]$ is valid and $\succ_{C_1}$ and $\succ_{C_2}$ are SPOs, then for all finite instances r:

$$\omega_{C_1}(\omega_{C_2}(r)) = \omega_{C_2}(\omega_{C_1}(r)) = \omega_{C_2}(r).$$

Semantic query optimization [Cho07b]

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Using information about integrity constraints to:

- eliminate redundant occurrences of winnow.
- make more efficient computation of winnow possible.

Eliminating redundancy

Given a set of integrity constraints F, ω_C is redundant w.r.t. F iff F implies the formula

$$\forall t_1, t_2.\ R(t_1) \wedge R(t_2) \Rightarrow t_1 \sim_C t_2.$$

Integrity constraints

Integrity constraints

Constraint-generating dependencies (CGD) [BCW99, ZO97]

$$\forall t_1. \ldots \forall t_n. \; [R(t_1) \wedge \cdots \wedge R(t_n) \wedge \gamma(t_1, \ldots t_n)] \Rightarrow \gamma'(t_1, \ldots t_n).$$

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CGD entailment

Decidable by reduction to the validity of $\forall$-formulas in the constraint theory (assuming the theory is decidable).

Top-K queries

Top-K queries

Scoring functions

- each tuple t in a relation has numeric scores $f_1(t), \ldots, f_m(t)$ assigned by numeric component scoring functions $f_1, \ldots, f_m$
- the aggregate score of t is $F(t) = E(f_1(t), \ldots, f_m(t))$ where E is a numeric-valued expression
- F is monotone if $E(x_1, \ldots, x_m) \leq E(y_1, \ldots, y_m)$ whenever $x_i \leq y_i$ for all i

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Top-K queries

- return K elements having top F-values in a database relation R
- query expressed in an extension of SQL:

```
SELECT *
FROM R
ORDER BY F DESC
LIMIT K
```

Top-K sets

Top-K sets

Definition

Given a scoring function F and a database relation r, s is a Top-K set if:

- $s \subseteq r$
- $|s| = \min(K, |r|)$
- $\forall t \in s.\ \forall t' \in r - s.\ F(t) \geq F(t')$

There may be more than one Top-K set: one is selected non-deterministically.

Example of Top-2

Example of Top-2

Relation $Car(Make, Year, Price)$

- component scoring functions:

$$f_1(m, y, p) = (y - 2005)$$

$$f_2(m, y, p) = (20000 - p)$$

- aggregate scoring function:
  $F(m, y, p) = 1000 \cdot f_1(m, y, p) + f_2(m, y, p)$

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Make	Year	Price	Aggregate score
mazda	2009	20000	4000
ford	2009	15000	9000
ford	2007	12000	10000

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Naive approaches

- sort, output the first K-tuples
- scan the input maintaining a priority queue of size K
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- the entire input does not need to be scanned...

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- ... provided additional data structures are available

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Naive approaches

- sort, output the first K-tuples
- scan the input maintaining a priority queue of size K
- ...

Better approaches

- the entire input does not need to be scanned...
- ... provided additional data structures are available
- variants of the threshold algorithm

Threshold algorithm (TA)[FLN03]

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Inputs

- a monotone scoring function $F(t) = E(f_1(t), \ldots, f_m(t))$
- lists S_i, $i = 1, \ldots, m$, each sorted on f_i (descending) and representing a different ranking of the same set of objects

1. For each list S_i in parallel, retrieve the current object w in sorted order:
   - (random access) for every $j \neq i$, retrieve $v_j = f_j(w)$ from the list S_j
   - if $d = E(v_1, \ldots, v_m)$ is among the highest K scores seen so far, remember w and d (ties broken arbitrarily)
2. Thresholding:
   - for each i, w_i is the last object seen under sorted access in S_i
   - if there are already K top-K objects with score at least equal to the threshold $T = E(f_1(w_1), \ldots, f_m(w_m))$, return collected objects sorted by F and terminate
   - otherwise, go to step 1.

TA in action

Aggregate score

$$F(t) = P_1(t) + P_2(t)$$

OID	P_1
5	50
1	35
3	30
2	20
4	10

OID	P_2
3	50
2	40
1	30
4	20
5	10

Priority queue

TA in action

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3	30
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5	10

Priority queue

T=100

TA in action

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Priority queue

5:60

T=100

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2	20
4	10

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4	20
5	10

Priority queue

3:80
5:60

T=100

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3:80
5:60

T=75

TA in action

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Priority queue

3:80
1:65
5:60

T=75

TA in action

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2	40
1	30
4	20
5	10

Priority queue

3:80
1:65
5:60
2:60

T=75

TA in databases

TA in databases

- objects: tuples of a single relation r
- single-attribute component scoring functions
- sorted list access implemented through indexes
- random access to all lists implemented by primary index access to r that retrieves entire tuples

Optimizing Top-K queries [LCIS05]

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Goals

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SELECT *
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Is there a better evaluation plan than materialize-then-sort?

Partial ranking of tuples

Partial ranking of tuples

Model

- set of component scoring functions $P = \{f_1, \ldots, f_m\}$ such that $f_i(t) \leq 1$ for all t
- aggregate scoring function $F(t) = E(f_1(t), \ldots, f_m(t))$
- how to rank intermediate tuples?

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Ranking principle

Given $P_0 \subseteq P$,

$$\bar{F}_{P_0}(t) = E(g_1(t), \ldots, g_m(t))$$

where

$$g_i(t) = \begin{cases} f_i(t) & \text{if } f_i \in P_0 \\ 1 & \text{otherwise} \end{cases}$$

Relations with rank

Relations with rank

Rank-relation R_{P_0}

- relation R
- monotone aggregate scoring function F (the same for all relations)
- set of component scoring functions $P_0 \subseteq P$
- order:

$$t_1 >_{R_{P_0}} t_2 \equiv \bar{F}_{P_0}(t_1) > \bar{F}_{P_0}(t_2)$$

Ranking intermediate results

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Operators

- rank operator μ_f: ranks tuples according to an additional component scoring function f
- standard relational algebra operators suitably extended to work on rank-relations

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Operator	Order
$\mu_f(R_{P_0})$	$t_1 >_{\mu_f(R_{P_0})} t_2 \equiv \bar{F}_{P_0 \cup \{f\}}(t_1) > \bar{F}_{P_0 \cup \{f\}}(t_2)$
$R_{P_1} \cap S_{P_2}$	$t_1 >_{R_{P_1} \cap S_{P_2}} t_2 \equiv \bar{F}_{P_1 \cup P_2}(t_1) > \bar{F}_{P_1 \cup P_2}(t_2)$

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```
SELECT *
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Unranked relation S

A	f_1	f_2	f_3
1	0.7	0.8	0.9
2	0.9	0.85	0.8
3	0.5	0.45	0.75

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Rank-relation $S_{\{f_1\}}$

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↑ $IndexScan_{f_1}$

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$\uparrow IndexScan_{f_1}$

$\xrightarrow{\mu_{f_2}}$

A	$\bar{F}_{\{f_1,f_2\}}$
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1	2.5
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Pipelined execution

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$\downarrow$ μ_{f_3}

A	$\bar{F}_{\{f_1,f_2,f_3\}}$
2	2.55
1	2.4

Algebraic laws for rank-relation operators

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Splitting for μ

$$R_{\{f_1,f_2,\ldots,f_m\}} \equiv \mu_{f_1}(\mu_{f_2}(\ldots(\mu_{f_m}(R))\ldots))$$

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Distributivity of μ over Cartesian product

$\mu_f(R_{P_1} \times S_{P_2}) \equiv \mu_f(R_{P_1}) \times S_{P_2}$ if f refers only to the attributes of R.

Part III

Preference management

Outline of Part III

3. Preference management
   - Preference modification

Preference modification

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Goal

Given a preference relation $\succ$ and additional preference or indifference information $\mathcal{I}$, construct a new preference relation $\succ'$ whose contents depend on $\succ$ and $\mathcal{I}$.

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General postulates

- fulfillment: the new information $\mathcal{I}$ should be completely incorporated into $\succ'$
- minimal change: $\succ$ should be changed as little as possible
- closure:
  - order-theoretic properties of $\succ$ should be preserved in $\succ'$ (SPO, WO)
  - finiteness or finite representability of $\succ$ should also be preserved in $\succ'$

Preference revision [Cho07a]

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Setting

- new information: revising preference relation $\succ_0$
- composition operator θ: union, prioritized or Pareto composition
- composition eliminates (some) preference conflicts
- additional assumptions: interval orders
- $\succ' = TC(\succ_0 \, \theta \succ)$ to guarantee SPO

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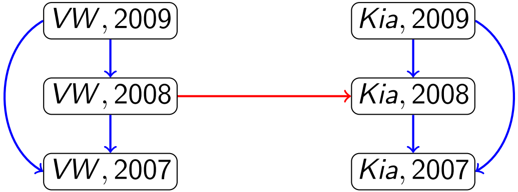

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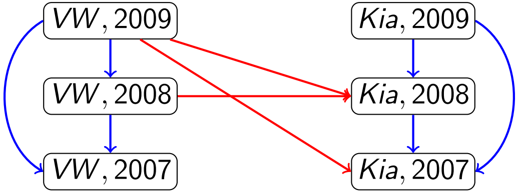

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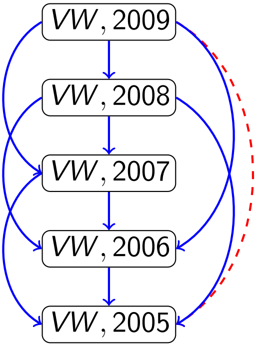

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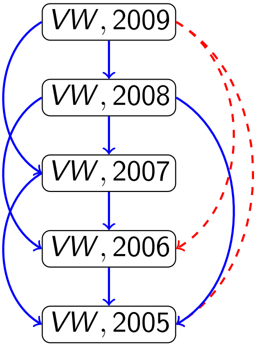

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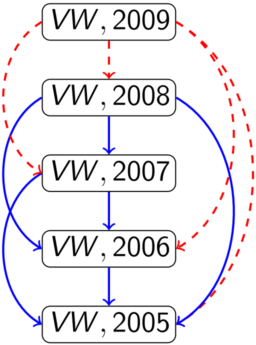

Substitutability [BGS06]

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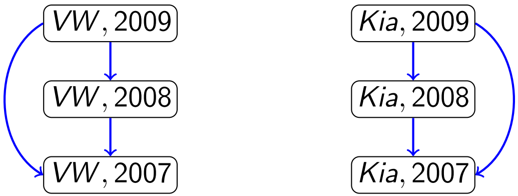

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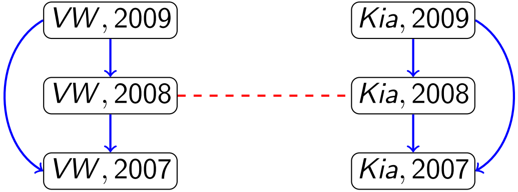

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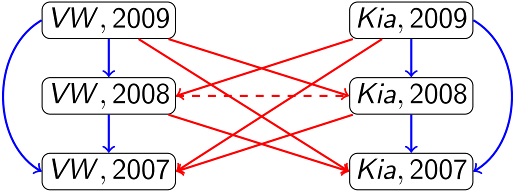

Part IV

Advanced topics

Outline of Part IV

Prospective research topics

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Definability

Given a preference relation $\succ_C$, how to construct a definition of a scoring function F representing $\succ_C$, if such a function exists?

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Extrinsic preference relations

Preference relations that are not fully defined by tuple contents:

$$x \succ y \equiv \exists n_1, n_2.\ Dissatisfied(x, n_1) \land Dissatisfied(y, n_2) \land n_1 < n_2.$$

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Preference relations that are not fully defined by tuple contents:

$$x \succ y \equiv \exists n_1, n_2.\ Dissatisfied(x, n_1) \wedge Dissatisfied(y, n_2) \wedge n_1 < n_2.$$

Incomplete preferences

- tuple scores and probabilities [SIC08, ZC08]
- uncertain tuple scores
- disjunctive preferences: $a \succ b \vee a \succ c$

Preference modification

- beyond revision and contraction: merging, arbitration,...
- general parametric framework?
- conflict resolution

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Variations

- preference and similarity: *"find the objects similar to one of the best objects"*

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Applications

- preference queries as decision components: workflows, event systems
- personalization of query results
- preference negotiation: applying contraction

Acknowledgments

Denis Mindolin
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Xi Zhang

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