Consistent Query Answering: The First Ten Years

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October 1, 2008

1 Motivation

2 Basics

Computing CQAMethods

Inconsistency and Incompleteness

6 Complexity

6 Variants of CQA

Conclusions

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Database instance D:

- a finite first-order structure
- the information about the world

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Integrity constraints Σ :

- first-order logic formulas
- the properties of the world

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Formula satisfaction in a first-order structure.

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Consistent database: D \models \Sigma
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Gates	Redmond	30M
Jobs	Cupertino	10M
Name \rightarrow City Salary		

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Whence Inconsistency?

Sources of inconsistency:

- integration of independent data sources with overlapping data
- time lag of updates (eventual consistency)
- unenforced integrity constraints



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Living with inconsistency?

- ignoring inconsistency
- modifying the schema
- exceptions to constraints.

► CQA

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Query results not reliable.

Decomposition into two relations:

- violators
- the rest

(De Bra, Paredaens [DBP83])



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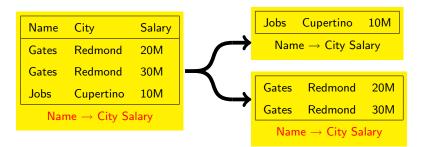
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• functional dependencies \rightsquigarrow denial constraints

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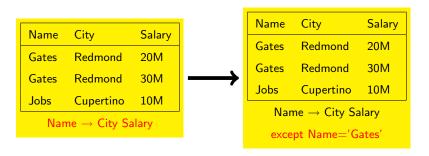
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Traditional view

- query results defined irrespective of integrity constraints
- query evaluation may be optimized in the presence of integrity constraints (semantic query optimization)

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Our view

- inconsistency leads to uncertainty
- query results may depend on integrity constraint satisfaction
- inconsistency may be eliminated (repairing) or tolerated (consistent query answering)

Database Repairs

Restoring consistency:

- insertion, deletion
- minimal change

Database Repairs

Restoring consistency:

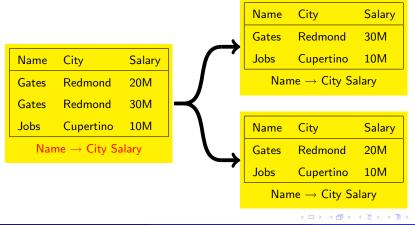
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Query answer obtained in every repair.

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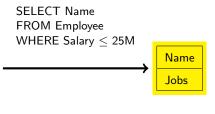
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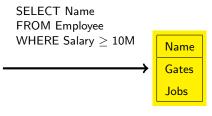
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What constitutes reliable (consistent) information in an inconsistent database.

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Algorithms

How to compute consistent information.

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Computational complexity analysis

- tractable vs. intractable classes of queries and integrity constraints
- tradeoffs: complexity vs. expressiveness.

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Implementation

• preferably using DBMS technology.

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Applications ??? Jan Chomicki () COA October 1, 2008 11 / 37

Repair D' of a database D w.r.t. the integrity constraints IC:

- D': over the same schema as D
- $D' \models IC$
- symmetric difference between D and D' is minimal.

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Consistent query answer to a query Q in D w.r.t. IC:

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Another incarnation of the idea of sure query answers [Lipski: TODS'79].



Belief revision

- semantically: repairing \equiv revising the database with integrity constraints
- consistent query answers \equiv counterfactual inference.

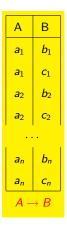
Logical inconsistency

- inconsistent database: database facts together with integrity constraints form an inconsistent set of formulas
- trivialization of reasoning does not occur because constraints are not used in relational query evaluation.

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Example relation R(A, B)

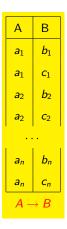
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- has 2ⁿ repairs.



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It is impractical to apply the definition of CQA directly.

Computing Consistent Query Answers

Query Rewriting

Given a query Q and a set of integrity constraints IC, build a query Q^{IC} such that for every database instance D

the set of answers to Q^{IC} in D = the set of consistent answers to Q in D w.r.t. IC.

Image: A math a math

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Representing all repairs

Given IC and D:

- **()** build a space-efficient representation of all repairs of D w.r.t. IC
- use this representation to answer (many) queries.

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- use this representation to answer (many) queries.

Logic programs

Given IC, D and Q:

- **()** build a logic program $P_{IC,D}$ whose models are the repairs of D w.r.t. IC
- 2 build a logic program P_Q expressing Q
- I use a logic programming system that computes the query atoms present in all models of P_{IC,D} ∪ P_Q.

Universal constraints

$$\forall . \neg A_1 \lor \cdots \lor \neg A_n \lor B_1 \lor \cdots \lor B_m$$

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Example

$$\forall . \neg Par(x) \lor Ma(x) \lor Fa(x)$$

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 $X \rightarrow Y$:

- a key dependency in F if Y = U
- a primary-key dependency: only one key exists

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 $R[X] \subseteq S[Y]$:

• a foreign key constraint if Y is a key of S

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Example foreign key constraint

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 $M[Manager] \subseteq M[Name]$

Building queries that compute CQAs

- relational calculus (algebra) \rightsquigarrow relational calculus (algebra)
- SQL → SQL
- leads to PTIME data complexity

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Query

Emp(x, y, z)

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Integrity constraint

$$\forall x, y, z, y', z'. \neg \textit{Emp}(x, y, z) \lor \neg \textit{Emp}(x, y', z') \lor z = z'$$

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Rewritten query

$$Emp(x, y, z) \land \forall y', z'. \neg Emp(x, y', z') \lor z = z'$$

(Arenas, Bertossi, Ch. [ABC99])

- Integrity constraints: binary universal
- Queries: conjunctions of literals (relational algebra: $\sigma, \times, -$)

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(Fuxman, Miller [FM07])

- Integrity constraints: primary key functional dependencies
- Queries: Cforest
 - a class of conjunctive queries $(\pi,\sigma, imes)$
 - no cycles
 - no non-key or non-full joins
 - no repeated relation symbols
 - no built-ins

• Generalization: conjunctive queries expressed as rooted rules (Wijsen [Wij07])

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SQL Rewriting

SQL query

SELECT Name FROM Emp WHERE Salary \geq 10K

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SQL rewritten query

SELECT e1.Name FROM Emp e1
WHERE e1.Salary ≥ 10K AND NOT EXISTS
 (SELECT * FROM EMPLOYEE e2
 WHERE e2.Name = e1.Name AND e2.Salary < 10K)</pre>

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(Fuxman, Fazli, Miller [FM05])

- ConQuer: a system for computing CQAs
- conjunctive (C_{forest}) and aggregation SQL queries
- databases can be annotated with consistency indicators
- tested on TPC-H queries and medium-size databases

Vertices

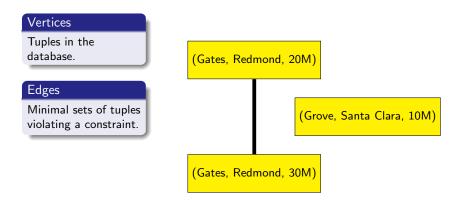
Tuples in the database.

(Gates, Redmond, 20M)

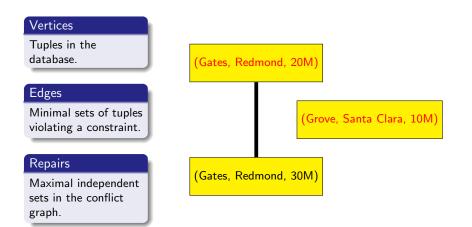
(Grove, Santa Clara, 10M)

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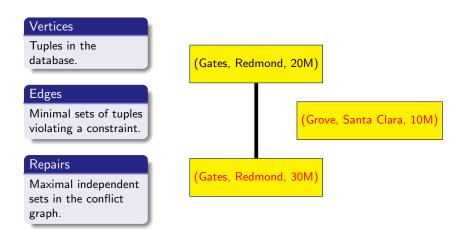
(Gates, Redmond, 30M)



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Algorithm HProver

INPUT: query Φ a disjunction of ground literals, conflict hypergraph *G* OUTPUT: is Φ false in some repair of *D* w.r.t. *IC*? ALGORITHM:

 \bigcirc find a consistent set of facts S such that

•
$$S \supseteq \{P_1(t_1), \ldots, P_m(t_m)\}$$

• for every fact $A \in \{P_{m+1}(t_{m+1}), \ldots, P_n(t_n)\}$: $A \notin D$ or there is an edge $E = \{A, B_1, \ldots, B_m\}$ in G and $S \supseteq \{B_1, \ldots, B_m\}$.

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(Ch., Marcinkowski, Staworko [CMS04])

- Hippo: a system for computing CQAs in PTIME
- quantifier-free queries and denial constraints
- only edges of the conflict hypergraph are kept in main memory
- optimization can eliminate many (sometimes all) database accesses in HProver
- tested for medium-size synthetic databases

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Logic programs

Specifying repairs as answer sets of logic programs

- (Arenas, Bertossi, Ch. [ABC03])
- (Greco, Greco, Zumpano [GGZ03])
- (Calì, Lembo, Rosati [CLR03b])

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Example

 $emp(x, y, z) \leftarrow emp_D(x, y, z), not \ dubious_emp(x, y, z).$ $dubious_emp(x, y, z) \leftarrow emp_D(x, y, z), emp(x, y', z'), y \neq y'.$ $dubious_emp(x, y, z) \leftarrow emp_D(x, y, z), emp(x, y', z'), z \neq z'.$

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 $dubious_emp(x, y, z) \leftarrow emp_D(x, y, z), emp(x, y', z'), z \neq z'.$

Answer sets

- {*emp*(*Gates*, *Redmond*, 20*M*), *emp*(*Grove*, *SantaClara*, 10*M*), ...}
- {*emp*(*Gates*, *Redmond*, 30*M*), *emp*(*Grove*, *SantaClara*, 10*M*),...}

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Logic Programs for computing CQAs

Logic Programs

- disjunction and classical negation
- checking whether an atom is in all answer sets is Π_2^p -complete
- dlv, smodels, ...

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- arbitrary first-order queries
- universal constraints
- approach unlikely to yield tractable cases

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Scope

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- universal constraints
- approach unlikely to yield tractable cases

INFOMIX (Eiter et al. [EFGL03])

- combines CQA with data integration (GAV)
- uses dlv for repair computations
- optimization techniques: localization, factorization
- tested on small-to-medium-size legacy databases

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There is a clear correspondence between repairs and possible worlds.

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Name	City	Salary	
Gates	Redmond	20M	
Gates	Redmond	30M	
Jobs	Cupertino	10M	
Name \rightarrow City Salary			

Image: A math a math

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Gates	Redmond	30M	
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Name \rightarrow City Salary			

 $Emp(Gates, Redmond, 20M) \lor Emp(Gates, Redmond, 30M)$

Image: A math a math

Emp(Jobs, Cupertino, 10M)

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Minimal models

Repairs vs. minimal models (Molinaro, Ch., Marcinkowski, 2008)

- For denial constraints, the set of repairs of an instance can be represented as the set of minimal models of a disjunctive database of at most exponential size
- The size of the disjunctive database is polynomial for a primary-key FD but may be exponential for two key FDs or one non-key FD.

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It is unlikely that new tractable cases of CQA will be obtained through the disjunctive database representation of the repairs:

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Conclusion

It is unlikely that new tractable cases of CQA will be obtained through the disjunctive database representation of the repairs:

but (Imielinski, van der Meyden, Vadaparty [IvdMV95])...

Name	City	Salary
Gates	Redmond	OR(20M,30M)
Jobs	Cupertino	10M

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Theorem (Ch., Marcinkowski [CM05a])

For primary-key functional dependencies and conjunctive queries, consistent query answering is data-complete for co-NP.

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For primary-key functional dependencies and conjunctive queries, consistent query answering is data-complete for co-NP.

Proof.

Membership: S is a repair iff $S \models IC$ and $W \not\models IC$ if $W = S \cup M$. Co-NP-hardness: reduction from MONOTONE 3-SAT.

- **9** Positive clauses $\beta_1 = \phi_1 \wedge \cdots \wedge \phi_m$, negative clauses $\beta_2 = \psi_{m+1} \wedge \cdots \wedge \psi_l$.
- 2 Database D contains two binary relations R(A, B) and S(A, B):
 - R(i, p) if variable p occurs in ϕ_i , i = 1, ..., m.
 - S(i, p) if variable p occurs in ψ_i , i = m + 1, ..., l.
- A is the primary key of both R and S.
- Query $Q \equiv \exists x, y, z. (R(x, y) \land S(z, y)).$
- (a) There is an assignment which satisfies $\beta_1 \wedge \beta_2$ iff there exists a repair in which Q is false.

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CQA

Q does not belong to C_{forest} .

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	Primary keys	Arbitrary keys	Denial	Universal
$\sigma, \times, -$				
$\sigma,\times,-,\cup$				
σ,π				
σ,π,\times				
$\sigma,\pi,\times,-,\cup$				

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	Primary keys	Arbitrary keys	Denial	Universal
$\sigma, \times, -$	PTIME	PTIME		PTIME: binary
$\sigma,\times,-,\cup$				
σ, π				
σ, π, \times				
$\sigma,\pi,\times,-,\cup$				

• (Arenas, Bertossi, Ch. [ABC99])

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$\sigma, \times, -$	PTIME	PTIME	PTIME	PTIME: binary
$\sigma,\times,-,\cup$	PTIME	PTIME	PTIME	
σ,π	PTIME	co-NPC	co-NPC	
σ, π, \times	co-NPC	co-NPC	co-NPC	
$\sigma,\pi,\times,-,\cup$	co-NPC	co-NPC	co-NPC	

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σ, π, \times	co-NPC	co-NPC	co-NPC	
	PTIME: C _{forest}			
$\sigma,\pi,\times,-,\cup$	co-NPC	co-NPC	co-NPC	

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- (Fuxman, Miller [FM07])

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$\sigma, \times, -$	PTIME	PTIME	PTIME	PTIME: binary
				Π_2^p -complete
$\sigma,\times,-,\cup$	PTIME	PTIME	PTIME	Π_2^p -complete
σ,π	PTIME	co-NPC	co-NPC	Π_2^p -complete
$\sigma, \pi, imes$	co-NPC	co-NPC	co-NPC	Π_2^p -complete
	PTIME: C _{forest}			
$\sigma,\pi,\times,-,\cup$	co-NPC	co-NPC	co-NPC	Π_2^p -complete

- (Arenas, Bertossi, Ch. [ABC99])
- (Ch., Marcinkowski [CM05a])
- (Fuxman, Miller [FM07])
- (Staworko, Ph.D., 2007), (Staworko, Ch., 2008):
 - quantifier-free queries
 - co-NPC for full TGDs and denial constraints
 - PTIME for acyclic full TGDs, join dependencies and denial constraints

The Explosion of Semantics

Tuple-based repairs

- asymmetric treatment of insertion and deletion:
 - repairs by minimal deletions only (Ch., Marcinkowski [CM05a]): data possibly incorrect but complete
 - repairs by minimal deletions and arbitrary insertions (Calì, Lembo, Rosati [CLR03a]): data possibly incorrect and incomplete
- minimal cardinality changes (Lopatenko, Bertossi [LB07])
- more in the pipeline...

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Attribute-based repairs

- (A) ground and non-ground repairs (Wijsen [Wij05])
- (B) project-join repairs (Wijsen [Wij06])
- (C) repairs minimizing Euclidean distance (Bertossi et al. [BBFL08])
- (D) repairs of minimum cost (Bohannon et al. [BFFR05])

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Computational complexity

- (A) and (B): similar to tuple based repairs
- (C) and (D): checking existence of a repair of cost < K NP-complete.

Tuple-based repairing leads to information loss.

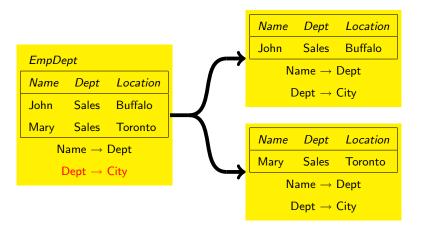
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Tuple-based repairing leads to information loss.

EmpDept				
Name	Dept	Location		
John	Sales	Buffalo		
Mary	Sales	Toronto		
$Name \to Dept$				
$Dept \to City$				

Image: A math a math

Tuple-based repairing leads to information loss.



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Repair the lossless join decomposition:

 $\pi_{Name,Dept}(EmpDept) \bowtie \pi_{Dept,Location}(EmpDept)$

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Attribute-based Repairs through Tuple-based Repairs (Wijsen [Wij06])

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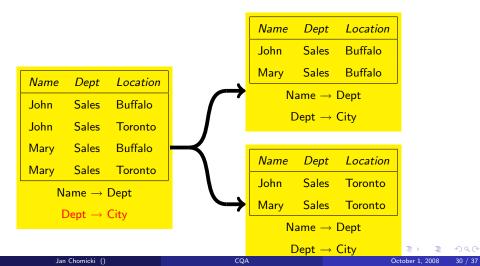
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(Andritsos, Fuxman, Miller [AFM06])

- potential duplicates identified and grouped into clusters
- worlds \approx repairs: one tuple from each cluster
- world probability: product of tuple probabilities
- clean answers: in the query result in some (supporting) world
- clean answer probability: sum of the probabilities of supporting worlds
 - consistent answer: clean answer with probability 1

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Salaries with probabilities

EmpProb				
Name	Salary	Prob		
Gates	20M	0.7		
Gates	30M	0.3		
Jobs	10M	0.5		
Jobs	20M	0.5		

Computing Clean Answers

SQL query

SELECT Name FROM EmpProb e WHERE e.Salary > 15M

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SELECT Name FROM EmpProb e WHERE e.Salary > 15M

SQL rewritten query

SELECT e.Name,SUM(e.Prob) FROM EmpProb e WHERE e.Salary > 15M GROUP BY e.Name

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$Name \to Salary$		

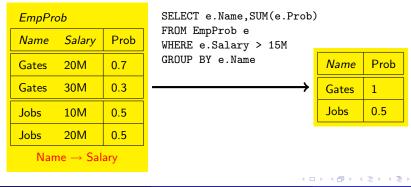
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Taking Stock: Good News

Technology

- practical methods for CQA for subsets of SQL:
 - restricted conjunctive/aggregation queries, primary/foreign-key constraints
 - quantifier-free queries, denial constraints/acyclic TGDs/JDs
 - LP-based approaches for expressive query/constraint languages
- (slow) emergence of generic techniques
- implemented in prototype systems
- tested on medium-size databases

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The CQA Community

- over 30 active researchers
- over 100 publications (since 1999)
- at least 8 doctoral dissertations in Europe and North America
- 2007 SIGMOD Doctoral Dissertation Award (Ariel Fuxman)
- overview papers [BC03, Ber06, Cho07, CM05b]

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Taking Stock: Initial Progress

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"Blending in" CQA

• data integration: tension between repairing and satisfying source-to-target dependencies

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"Blending in" CQA

 data integration: tension between repairing and satisfying source-to-target dependencies

Extensions

- uncertainty:
 - inconsistency leads to but cannot be reduced to uncertainty
 - repairs vs. possible worlds
 - probabilistic data
 - nulls: SQL conformance
- priorities:
 - preferred repairs
 - application: conflict resolution
- XML
 - notions of integrity constraint and repair
 - repair minimality based on tree edit distance?
- aggregate constraints

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Taking Stock: Largely Open Issues

Applications

- no deployed applications
- repairing vs. CQA: data and query characteristics
- heuristics for CQA and repairing

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CQA in context

- taming the semantic explosion
- CQA and data cleaning
- CQA and schema matching/mapping

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Foundations

- repair checking
- defining measures of consistency
- proving non-existence of rewritings

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Acknowledgments

Marcelo Arenas Alessandro Artale Leo Bertossi Loreto Bravo Andrea Calì Thomas Fiter Wenfei Fan Enrico Franconi Ariel Fuxman Gianluigi Greco Sergio Greco Phokion Kolaitis Domenico Lembo Maurizio Lenzerini Jerzy Marcinkowski Renée Miller Cristian Molinaro Riccardo Rosati Sławek Staworko David Toman Jef Wijsen

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Jan Chomicki ()

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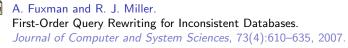
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