CSE 531 Homework Assignment 6

Due in class on Tuesday, Dec 04.

December 6, 2007

There are totally 6 problems, 10 points each. You should do them all. We will grade only 4 problems chosen at my discretion. If it so happens that you don't do one of the problems we don't grade, then no points will be deducted.

Note: you can only assume that one of the problems in the map given in the lecture notes is **NP**-complete.

Problem 1. Our CSE department currently has n students. Student i has taken a subset S_i of the set S of m courses that the department has ever offered. The DISJOINT COURSES problem asks: given a positive integer $k \leq m$, are there at least k students no two of whom have ever taken the same course?

Show that DISJOINT COURSES is **NP**-complete.

Problem 2. A boolean formula ϕ is said to be *no-negation* if it consists of a set of clauses which have no negations. For example,

$$\phi = (x_1 \vee x_2) \wedge (x_2 \vee x_3 \vee x_5) \wedge x_4$$

is a no-negation boolean formula. Certainly, setting all variables to TRUE will satisfy a no-negation formula. The NO-NEGATION SATISFIABILITY problem asks a more difficult question: given a no-negation boolean formula ϕ with n variables and m clauses, and a positive integer $k \leq n$, is there a truth assignment satisfying ϕ which sets at most k variables to be TRUE?

Show that NO-NEGATION BOOLEAN FORMULA is **NP**-complete.

Problem 3. Let $G = (L \cup R, E)$ be a bipartite graph. (Edges connect L and R.) An (a, b)-*subset* is defined to be a subset E' of edges such that *at most* a vertices in L are incident to some edges in E' and *at least* b vertices in R are incident to some edges in E'. The (a, b)-SUBSET PROBLEM is to decide if a given graph G has an (a, b)-subset.

Show that (a, b)-SUBSET is **NP**-complete.

Problem 4. An *alphabet* is a set of symbols. A *dictionary* is a set of words (i.e. strings) on the alphabet. A *document* is a collection of words from the dictionary. A word may occur more than once in a document.

The DOCUMENT COMPOSITION problem is defined as follows. We are given an alphabet and a dictionary on the alphabet. For each symbol s from the alphabet, we are also given a non-negative integer $f(s)$, interpreted as the frequency of the symbol s. The problem is to determine if there exists a document in which each symbol s appears exactly $f(s)$ times.

Show that DOCUMENT COMPOSITION is **NP**-complete.

Problem 5. The ZERO-CYCLE problem is defined as follows. An instance of the problem consists of a directed graph $G = (V, E)$ with integral edge weights w_e for each edge $e \in E$. Edge weights can be positive or negative or zero. The problem is to determine if G has a directed cycle whose edge weights sum up to zero.

Show that ZERO-CYCLE is **NP**-complete.

Problem 6. The COURSE ASSIGNMENT problem is defined as follows. There are n students, m courses, and p teachers. For every student i, course j, and teacher k, there is a given "satisfaction value" s_{ijk} which is a non-negative integer. A *course assignment* is defined as follows:

- A teacher can teach multiple courses, but a course can be taught by only one teacher. It's ok if some teacher(s) does not teach any course.
- A course can be attended by many students, but a student can only attend one course. It's ok if some course(s) does not have any student.
- Every student has to attend some course.

The total satisfaction value of a course assignment is the sum over all satisfaction values s_{ijk} for which student i attends course j taught by teacher k. (A teacher or a course which have no student do not contribute any satisfaction.) The question is: given a bound b, is there a course assignment with total satisfaction value at least b?

Show that the COURSE ASSIGNMENT problem is **NP**-complete.