The New York Review of Books

VOLUME 53, NUMBER 16 · OCTOBER 19, 2006

Review

Writing Nature's Greatest Book

By Freeman Dyson

The Best of All Possible Worlds: Mathematics and Destiny
by Ivar Ekeland
University of Chicago Press, 207 pp., $25.00

Ivar Ekeland has a Norwegian name and teaches at the University of British Columbia in Canada, but the style and spirit of his book are unmistakably French. The book is a rapid run through the history of the last four hundred years, seen through the eyes of a French mathematician. Mathematics appears as a unifying principle for history. Ekeland moves easily from mathematics to physics, biology, ethics, and philosophy. The central figure of his narrative is the French savant Pierre de Maupertuis (1698–1759), a man of many talents, who formulated the principle of least action in 1745 in a memoir with the title *The Laws of Motion and Rest Deduced from a Metaphysical Principle*. The principle of least action says that nature arranges all processes so as to minimize a quantity called action, which is a measure of the effort required to bring the processes to completion. The action of any mechanical motion is defined as the moving mass multiplied by the velocity and by the distance moved. Maupertuis was able to demonstrate mathematically that if a collection of objects moves in such a way as to make the total action as small as possible, then the movement obeys Newton's laws of motion. Thus the whole science of Newtonian mechanics follows from the principle of least action.

Maupertuis was dazzled by the beauty of his discovery. "How satisfying for the human spirit," he wrote, "to contemplate these laws, so beautiful and simple, which may be the only ones that the Creator and Ordainer of things has established in matter to sustain all phenomena of this visible world." He went on to identify action with evil, so that the principle of least action became a principle of maximum goodness. He concluded that God has ordered the universe so as to maximize goodness. The world that we live in is the best of all the possible worlds that God might have created. This simple principle unites science with history and morality. Mathematics is the key to the understanding of human destiny.

One of the contemporaries of Maupertuis was Voltaire, the great skeptic, who demolished Maupertuis's optimistic philosophy in a book with the title *Story of Doctor Akakia and the Native of Saint-Malo*. *Akakia* is Greek for "absence of evil," and the native of Saint-Malo is Maupertuis. "The native of Saint-Malo," Voltaire writes, "had long fallen a prey to a chronic sickness, which some call philotimia [Greek for love of honors] and others philocratia [Greek for love of power]." Voltaire's book sold well and Maupertuis's day of glory ended. After Maupertuis died, Voltaire made him posthumously ridiculous by writing the novel *Candide*, in which Maupertuis appears as the optimistic philosopher Pangloss, wandering from one disaster to another but unshaken in his belief that "all is well that ends well in the best of all possible worlds."

Maupertuis was in fact no Pangloss. He spent only a small part of his time as an optimistic philosopher. He was also a brilliant scientist and a capable administrator. He became famous as a young man for leading an expedition to Lapland to measure the shape of the earth at high latitude. His measurements were accurate enough to prove that the earth is not a perfect sphere but an ellipsoid, flattened at the poles as Newton predicted as a consequence of its rotation. This confirmation of Newton's theory was historically important, since up to that time Newtonian physics was not widely known or accepted in France. Maupertuis also learned to travel on skis in Lapland, and brought home with him the first pair of skis that had ever been seen in France. For many years after the Lapland expedition, he was one of the most active members of the French Academy of Sciences. When King Frederick the Great of Prussia founded his own Academy of Sciences in Berlin, he invited Maupertuis to be the first president. Maupertuis spent the rest of his life in Berlin, successfully launching and running the Prussian Academy. Voltaire hated King Frederick, and Maupertuis's friendship with the King gave Voltaire another reason to hate and belittle Maupertuis.

Ekeland's sketch of history is divided into two parts, before

Maupertuis and after Maupertuis. Before Maupertuis, the two chief characters are Galileo and Descartes. Galileo started modern science by using the pendulum as a tool to make accurate measurements of time. Ancient Greek science was based on geometry, measuring space but not time. Archimedes understood statics but did not understand dynamics. Galileo with his pendulum and his falling weights made the decisive step from a static to a dynamic view of nature. He introduced time as a quantity accessible to mathematical analysis. He said, "Nature's great book is written in mathematical symbols." That remark of Galileo was the lever that moved the world into the modern era of scientific understanding.

After Galileo came René Descartes, a great mathematician and a great philosopher but not yet a great scientist. Descartes took to heart Galileo's insight that mathematics is the language that nature speaks. He tried to deduce the laws of nature from the laws of mathematics by pure reason alone. He did not listen to another statement of Galileo, that nature answers questions that we ask by doing experiments. Descartes held experimental results in low esteem, thinking them less trustworthy than logic. His was a normative science, telling nature what it was supposed to do, and not an experimental science, investigating what nature was actually doing. Descartes published in 1637 his great work, *A Discourse on the Method of Rightly Conducting the Reason and of Seeking Truth in the Sciences*. He describes a scientific method that is broad enough to deal with moral as well as with physical problems. "I showed what the laws of nature were," he wrote,

> and without basing my arguments on any principle other than the infinite perfections of God, I tried to demonstrate all those laws about which we could have any doubt, and to show that they are such that, even if God created many worlds, there could not be any in which they failed to be observed.

Ekeland concludes that Descartes's method "has been used in science with tremendous success, and there is no reason why it should not be as useful in philosophy, or in trying to establish some principles by which to guide our collective and individual lives." Unfortunately, the Cartesian way of doing science with minimum recourse to experimentation led him into bad mistakes. From his philosophical principle that nature abhors a vacuum, he was led to deduce that the space around the planets is filled with enormous vortices, or whirling masses, and that the pressure of the vortices confines the planets to their orbits and pushes them on their way. This theory of planetary motions was generally accepted in France as a preferable alternative to Newton's theory of universal gravitation. Descartes also deduced that the rotating earth creates another enormous vortex that squeezes the earth into the shape of an American or rugby football. According to Descartes, the earth should be an ellipsoid elongated at the poles, instead of being flattened as predicted by Newton. Maupertuis's measurements in Lapland proved Newton right and Descartes wrong.

Ekeland's history continues after Maupertuis with a couple of great mathematicians—Joseph-Louis Lagrange and Henri Poincaré, who used the ideas of Maupertuis to build a grand edifice of classical dynamics. Poincaré, in the late nineteenth century, discovered chaos, a general property of dynamical systems that makes their behavior unpredictable over long times. He discovered that almost all complicated dynamical systems are chaotic. In particular, the orbital motions of planetary systems with more than two planets, and the fluid motions of atmospheres or oceans, are likely to be chaotic. The discovery of chaos opened a new chapter in the history of astronomy and meteorology, as well as in the history of mathematics.

After his discussion of Poincaré, Ekeland devotes chapters to biology and ethics, with backward glances to establish connections with Maupertuis. In biology, the guiding principle of evolution is the survival of the fittest. Darwin's notion of nature selecting a population with maximum fitness resembles Maupertuis's notion of God selecting a universe with maximum goodness. Darwin himself understood that fitness is not the same as goodness, but other evolutionary thinkers such as Herbert Spencer allowed the distinction between fitness and goodness to be blurred. Darwin rarely used the word "evolution," which Spencer introduced into biology. Darwin preferred to speak of "descent with variation," emphasizing the fact that variations are random and not usually progressive.

In ethics, the problem of optimization is even more tricky. Ekeland begins his discussion of ethics with Jean-Jacques Rousseau, the philosopher of the French Enlightenment, whose ideas prepared the way for the revolution of 1789. Rousseau believed that human beings were naturally virtuous and wise. They needed only to be set free from tyrannical governments, and then they would order their affairs harmoniously. A democratic government, responsive to the will of a free people, would make sure that everyone was treated fairly. Before the revolution could put these ideas to a practical test, some theoretical difficulties were raised by the Marquis de Condorcet, who for the first time used mathematics to model human behavior. The marquis discovered a logical inconsistency known as Condorcet's paradox, which demonstrates that an assembly ruling by majority vote may make decisions that are logically incompatible. For example, if three candidates A, B, C are running for a job to be filled by majority vote, it is possible that a majority prefers A to B, another majority prefers B to C, and a third majority prefers C to A. Then the result of the election will depend on the order in which the votes are taken. Another learned academician, the Chevalier de Borda, devised a system of preferential voting for election of members to the French Academy of Sciences. The de Borda scheme avoided the Condorcet paradox, but led to another paradox that could be exploited by unscrupulous politicians to win elections. It turned out that no system of voting is free from mathematical paradoxes. And the revolution, when it came, brought a quarter-century of death and destruction instead of the peace and harmony that Rousseau had promised.

To sum up the lessons to be learned from history, Ekeland writes:

> We have now reached the end of our journey. It started in the world of the Renaissance, impregnated with Christian values.... The laws of nature then are simply the rules God followed when creating the world, and the purpose of science is to recover them from observations. There is then also a deeper science, which is to seek the purpose God himself had in creating the world. This is what Maupertuis, in a glorious moment, thought he had achieved, thereby reconciling forever science and religion, both being the quest for God's will, in the physical world and in the moral one. Our journey ends in a world where God has receded, leaving humankind alone in a world not of its choosing.

While reading this account, I became more and more intrigued by the question how a Norwegian working in Canada acquired a view of mathematics and of history that is so quintessentially French. The characters in his story are mostly French, and the dominant role of mathematics in their thinking is a hallmark of French culture. Nowhere else except in France do mathematicians command such respect. As soon as I consulted Google, I found the solution to the mystery. In spite of his Norwegian name, Ekeland is French. Born in Paris, educated at the historic École Normale Supérieure, professor at the University of Paris–Dauphine, and subsequently president of the university, he is a charter member of the French academic establishment. His books were mostly written in French before being published in other languages. This book is a translation of a book with the same title published in French in the year 2000, revised and brought up to date for English-speaking readers. It gives us a vivid picture of human history and destiny as seen through the eyes of a senior academic trained in the French educational system.

There is at least one Frenchman who does not share Ekeland's view of the world. Pierre de Gennes is a brilliant French physicist who won a Nobel Prize in 1991 for understanding the behavior of squishy materials on the borderland between liquid and solid. He called the things that he studied "soft matter." After the Nobel Prize made him a French national hero, he was inundated with invitations to visit high schools and inspire the students to follow in his footsteps. He accepted the invitations and spent a year and a half as a traveling guru, explaining science to the kids. He enjoyed the contact with young people so much that he turned his talks into a book, *Fragile Objects: Soft Matter, Hard Science, and the Thrill of Discovery*. The book was translated into English and published by Springer-Verlag in 1996. It describes in simple words how the science of soft matter explains the behavior of ordinary materials such as soap, glue, ink, rubber, and flesh and blood that children encounter in their everyday lives. De Gennes's talks were aimed at the average child, not at the talented few who might become professional scientists. His book is well pitched to give average readers a practical understanding of how science works.

At the end of his book, de Gennes adds a few chapters aimed not at the children but at their teachers. One of these chapters, with the title "The Imperialism of Mathematics," is a diatribe against the dominance of mathematics in the French educational system. He writes:

> Whenever an entrance examination is instituted in a scientific discipline, it invariably becomes an exercise in mathematics.... Why is there such a focus on mathematics? In reality, the trend toward mathematization turns our graduates, our future engineers, into hemiplegics.... They may have learned to master certain tools, to prepare reports, but they will suffer from crippling weaknesses in observation, manual skills, common sense, and sociability.

De Gennes is not a typical French intellectual. He mixes theory with experiment, and prefers concrete objects to abstract ideas. In his research and in his teaching, he fights against the imperialism of mathematics.

In America we have the opposite situation. Our children study a variety of subjects without much formal discipline, and most of them remain mathematically illiterate. It is good for us to be reminded that different countries have profoundly different cultures and different virtues and vices. The imperialism of mathematics is difficult for Americans to imagine, but for France it is a real problem. If American children could learn more mathematics and French children less, both countries would benefit. Americans should not be misled by de Gennes's diatribe into thinking that we have nothing to learn from France. He describes eloquently the vices of the French educational establishment. He does not emphasize its virtues. The most important virtue of the French system is the strict discipline that it imposes. Every child and every student must meet rigid standards of knowledge and skill. De Gennes takes for granted the fact that the children he is talking to are literate and have a firm grasp of elementary mathematics. Americans should ask themselves why such a standard of literary and mathematical competence cannot be taken for granted in America.

Ekeland does not entirely exclude people who were not French from his narrative. He recognizes the great contributions of Galileo, Newton, Euler, and Darwin to the development of modern science, and the great contributions of the historians Thucydides and Guicciardini to the understanding of human destiny. Some of the most illuminating passages in the book are quotations from Thucydides and Guicciardini, both of them generals who fought on the losing side in catastrophic wars and then wrote their histories to teach whatever bitter lessons posterity might learn from their defeat. Both of them saw tragedy arising not from implacable fate but from human folly and unlucky accidents. With wiser leaders, mistakes might have been avoided and tragedy averted. The worst mistakes are mistakes of overconfidence, made by arrogant leaders who do not respect the skill of their enemies or the vagaries of chance. For the American edition of his book, Ekeland has inserted some acid remarks about arrogance and overconfidence displayed in recent actions of the American government.

A different book about the cultural history of the last four hundred years might have been written by a different Ekeland who was educated in the Anglo-American tradition instead of the French. I call the imaginary Ekeland Akeland, and I assume that Akeland is as strongly biased toward English as Ekeland is biased toward French. For Akeland, modern science still begins with Galileo, but then continues with Francis Bacon instead of René Descartes. Bacon was three years older than Galileo and thirty-five years older than Descartes. Bacon pushed English science as strongly in the direction of experiment as Descartes pushed French science in the direction of theory. Bacon had a low opinion of theory. He wrote: "The logic now in use serves rather to fix and give stability to the errors which have their foundation in commonly received notions than to help the search after truth." Bacon preached humility toward nature as the only way to arrive at truth: "Man, being the servant and interpreter of Nature, can do and understand so much and so much only as he has observed in fact or in thought of the course of Nature; beyond this he neither knows anything nor can do anything." He had a grand vision of the future of science but a modest view of the science of his own time: "For though it be true that I am principally in pursuit of works and the active department of the sciences, yet I wait for harvest-time, and do not attempt to mow the moss or to reap the green corn." He did not live to see the harvest of discoveries that began thirty-four years after his death when the Royal Society of London was founded. He died while the corn was still green and Descartes had not yet started to mow the moss.

In Akeland's version of history, the scientist who personifies eighteenth-century enlightenment is Benjamin Franklin rather than the Marquis de Maupertuis. Instead of the mathematicians Lagrange and Poincaré, the scientists who bring us into the modern world are the nineteenth-century British physicists Michael Faraday and James Clerk Maxwell, who set out the basic laws of electricity and magnetism. Bacon, Franklin, Faraday, and Maxwell, the chief characters in Akeland's narrative, are nowhere mentioned by Ekeland. Likewise, Akeland fails to mention Descartes, Maupertuis, Lagrange, and Poincaré. His main theme is the emergence of electricity in the eighteenth century as the growing point of science. Electricity was a product of purely Baconian science, emerging from unexpected observations of nature rather than from mathematical deduction.

Ekeland's book puts mathematical optimization at the focus of history. Optimization means choosing the best out of a set of alternatives. Mathematical optimization means using mathematics to make the choice. Maupertuis is the central character of the history because he claimed that the universe is mathematically optimized. Akeland's book has the opposite emphasis. For Akeland, things are more important than theorems. Experiments are more important than mathematics. The great scientific achievement of the Age of Enlightenment was the experimental study of electricity. Electricity was the driving force of science for two hundred years, from the death of Newton to the rise of molecular biology. Electricity also enlarged the scope of science, moving out from the logical and mechanical universe of Newton into the color and variety of the modern world. The biologist Stephen Jay Gould formulated the philosophical principle that Akeland borrows for the title of his book: "We are the offspring of history and must establish our own paths in this most diverse and interesting of conceivable universes." Instead of mathematical optimization, Akeland postulates maximum diversity as the governing principle of the universe. His title is *The Most Interesting of All Possible Worlds: Electricity and Destiny*.

Franklin had no theoretical understanding of electricity. Electricity was outside the Newtonian domain of mechanics and gravitation that constituted the theoretical science of his time. Franklin explored electricity because it was a part of nature that nobody understood. Without pretending to understand electricity, he learned how to control it. His invention of the lightning conductor made him world-famous and earned him a warm welcome when he came to live in France. He came to France too late to meet with Maupertuis. If they had met, they would have found that they had much in common. Franklin was only eight years younger than Maupertuis. Both were good organizers as well as good scientists. Franklin was organizing the American Philosophical Society in

Philadelphia while Maupertuis was organizing the Prussian Academy in Berlin. Both were gentlemen of the Enlightenment, adventurers and travelers in an age when travel was slow and arduous. Both were by temperament optimists, but neither was a Pangloss. The only serious difference between them was that Maupertuis was a mathematician and Franklin was an experimenter.

The next pair of characters in Ekeland and Akeland's histories were Lagrange in France and Faraday in England. They lived in different centuries and had less in common than Maupertuis and Franklin. They were extreme examples of Cartesian and Baconian scientists. Faraday explored the new worlds of electricity and magnetism, chemistry and metallurgy, pushing into unknown territory far ahead of any theoretical understanding. Lagrange (1736–1813) created the science of analytical mechanics, an abstract mathematical framework that included all the results of Newtonian dynamics as special cases. Each was master of his trade, but theirs were very different trades. By unifying Newton's ideas into a single scheme, Lagrange left the world simpler than he found it. By discovering a host of unexpected new phenomena, Faraday (1791–1867) left the world more complicated than he found it. Lagrange was a unifier; Faraday was a diversifier. Although Lagrange's great work was published three years before Faraday was born, Faraday never read it and never felt a need for it. All the mathematics that Faraday needed was elementary arithmetic and a little algebra.

The histories of Ekeland and Akeland begin to diverge with Maupertuis and Franklin and reach a point of maximum divergence with Lagrange and Faraday. With the last pair of characters, Poincaré and Maxwell, the histories converge. Poincaré (1854–1912) was a mathematician with a taste for diversity. He was interested in the new science of electromagnetism as well as the old science of mechanics, and he discovered in the dynamics of stars and planets a variety of chaotic motions that Lagrange never dreamed of. Maxwell (1831–1879) was a physicist with a passion for unification. Starting from the observations of Faraday, he discovered the equations that unify the theories of electricity and magnetism and light into a mathematical structure as elegant as Lagrange's mechanics. The convergence of Ekeland and Akeland became complete when Poincaré explored the group of symmetries of the Maxwell equations, the group that is now known to physicists as the Poincaré Group. Maxwell and Poincaré together prepared the way that led Einstein to the new world of relativity.

The real Ekeland and the fictitious Akeland are teaching us a simple lesson. Each of them gives us a slanted and partial view of history. The true history of modern science must include both of them. Modern science started its rapid growth in the seventeenth century, taking its aims and methods not from Descartes alone and not from Bacon alone but from the cross-fertilization of Cartesian and Baconian ideas. Isaac Newton, the greatest figure in the history of the physical sciences, was an intimate mixture of Descartes and Bacon. He was Baconian in his study of optics, when he separated white light into its colored components and invented his reflecting telescope. He was Cartesian when he wrote his *Principia Mathematica*, deducing the system of the world from a logical sequence of mathematical propositions. He cleverly used a Cartesian style of argument, together with a Baconian knowledge of planetary motions, to demolish Descartes's cosmology of vortices in space.

In a true history of science, mathematics and electricity make equal contributions to human destiny. Our world may be the best of all possible worlds and may be the most interesting. Both possibilities are open. Our destiny depends on choices that we have not yet made, probably concerned more with biology—and particularly with our incipient understanding of the human brain—than with mathematics or electricity.