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LANDSCAPE OF COMPUTATIONAL COMPLEXITY

Spring 2008

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A complete language for EXPSPACE: PIM, "Polynomial Ideal Membership"—the simplest natural completeness level that is known not to have polynomial-size circuits.

PIM
EXPSPACE
Succinct 3SAT
NEXP
co-NEXP
nxn Chess
EXP
QBF
PSPACE
Σ^p_2
Π^p_2
TAUT
SAT
NLIN
NTIME [n^2]
NP
co-NP
FACT
P
CVP
GAP
NL
UGAP
L
WS_5
REG
REG ≠ L
NP
P
DLIN
etc
DTIME [n^3]
DTIME [n^2]

For any fixed k, there is a problem in this intersection that can NOT be solved by circuits of size $O(n^k)$

A Deterministic and Nondeterministic Time Hierarchies Within NP

Unknown but Commonly Believed:

- L ≠ NL ..... L ≠ PH
- P ≠ NP ∩ co-NP ..... P ≠ PSPACE
- NP ≠ $\Sigma^p_2 \cap \Pi^p_2$ ..... NP ≠ EXP

Best Known Separations:

- $AC^0 \subset ACC^0 \subset PP$, also $TC^0 \subset PP$
- $NC^1 \subset PSPACE$, ... , $NL \subset PSPACE$
- $P \subset EXP$, $NP \subset NEXP$
- $PSPACE \subset EXPSPACE$

L
WS_5
NC^1
TC^0
PARITY
ACC^0
AC^0
REG
$AC^0 \neq ACC^0$

C Low-Level Classes

WS_5, the word problem for the symmetric group S_5, is a regular language that is complete for NC^1 under AC^0 many-one reductions.

B Complexity "Main Sequence"

Arithmetical Hierarchy (AH)

Σ_n
$K^{(2)}$
TOT
Π_n
Σ_2
REC^{RE}
Π_2
$= \exists\forall\cdot REC$
$= \forall\exists\cdot REC$
K
D
RE
co-RE
REC
$= \exists\cdot REC$
$= \forall\cdot REC$

TOT = {M_e : M_e is total, i.e. halts for all inputs}

D = {Turing machines M_e: M_e does not accept e} = the complement of K. ("Diagonal Language")

Polynomial Hierarchy (PH)

Σ^p_2
Π^p_2
$= \exists\forall^{poly}.P$
$= \forall\exists^{poly}.P$
P^{NP}
NP
co-NP
P
$= \exists^{poly}.P$
$= \forall^{poly}.P$

The levels of AH and PH are analogous, except that we believe $NP \cap co\text{-}NP \neq P$ and $\Sigma^p_2 \cap \Pi^p_2 \neq P^{NP}$, which stand in contrast to $RE \cap co\text{-}RE = REC$ and $\Sigma_2 \cap \Pi_2 = REC^{RE}$

D Analogy between Arithmetical and Polynomial Hierarchies

BQP: Bounded-Error Quantum Polynomial Time. Believed larger than P since it has FACTORING, but not believed to contain NP.

BPP: Bounded-Error Probabilistic Polynomial Time. Many believe BPP = P.

QBF
PP Probabilistic Polynomial Time
MAJ-SAT
PSPACE
Σ^p_2
Π^p_2
NP
co-NP
FACT
P
BQP
BPP

FACTORING is not believed to be complete for BQP or for NP ∩ co-NP.

E Realm of Feasibility?