Name ___SOLUTION SET____

CSE 241 Digital Systems October 12, 2009

Hourly Exam #1

1. Let A=1010.1 and B=11.1

*(a) Find C=A*B for A,B,C all base-2*

*(b) Find C=A*B for A,B,C all base-5*

(c) Find D=A/B for A,B,D all base-2

```
1(a)        1010.1      1(b)      1010.1
              11.1                  11.1
            ------                ------
            1010 1                1010 1
           10101                 10101
          10101___              10101___
         100100.11              11212.11
```

```
1(c)
                  1 1
              -------
         11.1|1010.1
                111
                ---
                 11 1
                 11 1
                 ----
                    0
```

1(a) ________100100.11________

1(b)________11212.11________

1(c)________11________

2(a) Find X-Y where X=0s218 and Y=0s219 are both duodecimal (base-12) numbers. Use any method, but show your result as a base-12 complement.

2(b) A Binary-Coded Duodecimal Code (BCDD code) encodes base-12 digits as n-bit binary numbers. Design a BCDD using the smallest n such that you can detect all 1-bit errors. Tell what n is for your code, show how you code the duodecimal alphabet 0,1,2,3,…,9,A,B and how you detect if there has been a 1-bit error.

11-s complement of 0s219 = 1s9A2, so 12-s complement is 1s9A3

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 0s218
+1s9A3
 1sBBB
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(Check: this is $BBB_{(12)}$ units from $-1000_{(12)}$, ie 218-219=-1)

2(a) 1sBBB

2(b)

Using 4-bit binary numbers we can encode duodecimal digits in counting order, ie $0_{(12)}=0000_{(2)}$, $1_{(12)}=0001_{(2)}$,… $B_{(12)}=1011_{(2)}$. By adding a 5th bit we can detect 1-bit errors, for instance by making the MSB an even parity bit. So if a code group comes in odd-parity, eg. 01011, we have detected an error. Then n=5 and the final code is:

base-12 digit	code group
0	00000
1	10001
2	10010
3	00011
…	…
A	01010
B	11011

Not enough room between the 12 code groups to do it using just n=4.

3. *f(x,y,z) = (x'+y)(y'+z')+(xz)'*

(a) Sketch the maxterm realization (circuit diagram) using AND and OR gates. Assume double-rail logic.

(b) Repeat (a) using only NAND gates (a-form and b-form ok).

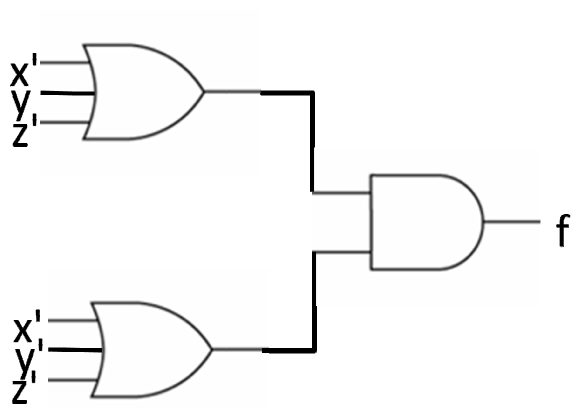
(a) f = x'y'+x'z'+0+yz'+x'+z'
= x'+z' (by absorption)

So truth table for f is

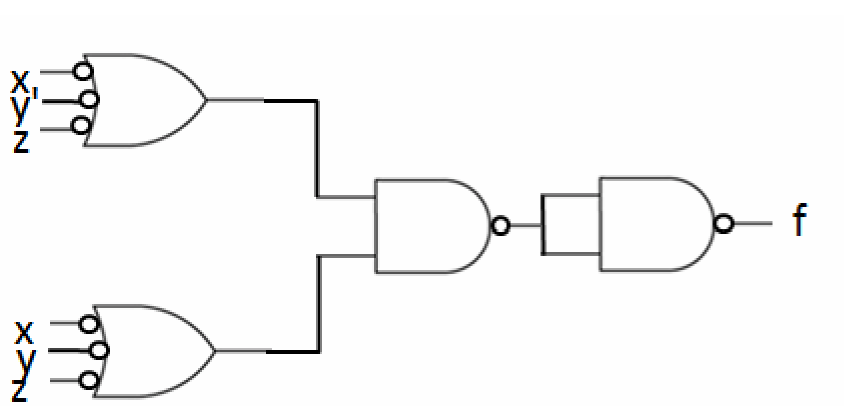
x	y	z	x'+z'
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	**0**	**1**	**0**
1	1	0	1
1	**1**	**1**	**0**

Use the zero-rows for maxterm

3(a)



3(b)



4. (a) Prove that no element x of any Boolean algebra can be its own complement x', ie $x \neq x'$ for all elements x of all Boolean algebras. You need not specify warrants in the steps of your proof. *Hint: consider proof by contradiction.*

(b) Use the Method of Perfect Induction to prove the Absorption Theorem $x+xy=x$ for the Boolean Algebra $B=\{0,1,2,3\}$ with + and * operations defined in Tables A and B. For your convenience, a blank truth table is shown on the right. Explain briefly in the space below Tables A and B why your work proves the theorem. *Note: parts (a) and (b) are not related.*

4(a)

Assume the contrary, that x=x'. We will show a contradiction. Start with

```
        x+x'=1      P5(a) Existence of Complements

Then    x+x =1      By assumption that x=x'
Or        x =1      By Idempotent Law
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So our assumption x=x' implies that x=1.

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But       x*x'=0    P5(b) Existence of Complements
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fails to be true if x and x' are both 1 since 1*1=1 by Idempotent Law. Since assuming x=x' leads to a falsehood, it must be that x≠x'.

4(b)

+	0	1	2	3
0	0	1	2	3
1	1	1	3	3
2	2	3	2	3
3	3	3	3	3

Table A: + operation

*	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

Table B: * operation

x	*y*	*xy*	*x+xy*
0	0	0	0
0	1	0	0
0	2	0	0
0	3	0	0
1	0	0	1
1	1	1	1
1	2	0	1
1	3	1	1
2	0	0	2
2	1	0	2
2	2	2	2
2	3	2	2
3	0	0	3
3	1	1	3
3	2	2	3
3	3	3	3

Since the first and last columns of the truth table on the right are identical, then x=x+xy for all values of x and y. Hence the theorem x=x+xy is true (for the algebra that the truth table was constructed for).