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CSE 241 Digital Systems Apr 6, 2011

Hourly Exam #2 Solution Set

Instructions: Write your name on the top of each sheet. Show all work in the space provided. No calculators or other electronic devices allowed. 50 min closed book.

$$f(w,x,y,z)=x'+z'+w'xyz$$

1(a) Sketch the Karnaugh Map for f, showing the prime implicant subcubes, and write the product term for each prime implicant of f.

		yz			
		00	01	11	10
wx	00				
	01				
	11				
	10				

There are three prime implicants: 2 of size 8 (black, blue), one of size 4 (red).

		yz			
		00	01	11	10
wx	00	1	1	1	1
	01	1		1	1
	11	1			1
	10	1	1	1	1

The product terms for the two of size 8 are z′ and x′, the other is w′y.

(b) Find the minimal sum. Show all work.

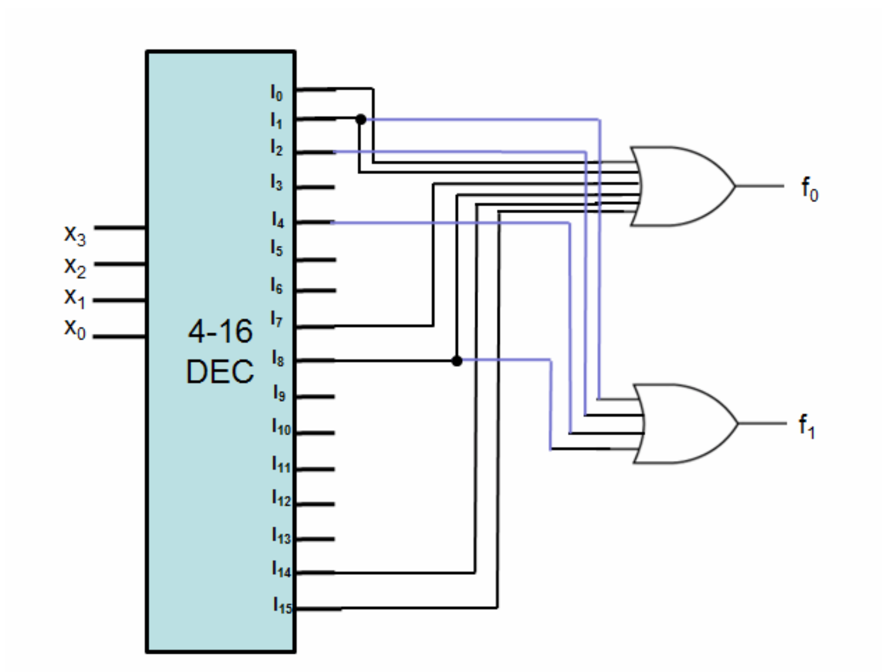
All three prime implicants contain essential 1-cells, so all are irredundant. Thus the only irredundant sum uses all three of them, and that must be minimal.

f(x,y,z)= x′+z′+w′y

2. We require a circuit with 4 inputs $x_0,..,x_3$ and two outputs f_0 and f_1. f_0 should equal 1 if and only if the binary number "$x_3x_2x_1x_0$" has three touching bits which are the same. So for instance the inputs "$x_3x_2x_1x_0$"="1000" should yield f_0=1, but "$x_3x_2x_1x_0$"="0100" should give f_0=0. Also, f_1 should equal 1 if and only if one of the four inputs is 1 and the rest are 0.

(a) Realize this circuit using the 4-16 DEC shown. Complete the circuit diagram wiring in OR gates as needed, and labelling the outputs f_0 and f_1.

The minterms for f0 are 0000,0001,1000,0111,1110,1111 which are outputs 0,1,8,7,14,15. So these outputs must be OR'ed to get f_0. The minterms for f_1 are 0001,0010,0100,1000 which are outputs 1,2,4,8. The circuit is shown below.



(b) Is the circuit constructed as in (a) above <u>minimal for any reasonable cost</u>? Justify your answer.

No it is not minimal. There are 8 unused outputs, each of which is attached to an AND gate within the DEC. These gats can be eliminated to reduce the gate count. So it is clearly not minimal.

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3.

	0001	*0101*	*0110*	*0111*	*1011*	*1100*	*1101*	*1111*
A	*x*	*x*						
B			*x*	*x*				
C					*x*			*x*
D						*x*	*x*	
E		*x*		*x*			*x*	*x*

The Prime Implicant Table for a Boolean function f(w,x,y,z) is shown above. The rows are the prime implicants, the columns are the minterms.

(a) Use the Petrick method to find all irredundant sums. Express your answer in terms of A, B, C, D, E.

p(A,B,C,D,E)= A(A+E)B(B+E)C(C+E)D(D+E)
= ABCD (Absorption Law)

So the only irredundant sum is A+B+C+D where A,B,C,D are the prime implicants.

(b) Write the minimal sum in terms of f's literals (w,x,y,z and their complements).

Since there is only one irredundant sum it must be the minimal sum. Looking at the Prime Implicant Table, A covers the minterms 0001 and 0101, which is w'y'z. Similarly, B covers 0110 and 0111, so B=w'xy, C covers 1011 and 1111 so C=wyz and D covers 1100 and 1101, so D=wxy'. Then the minimal sum is

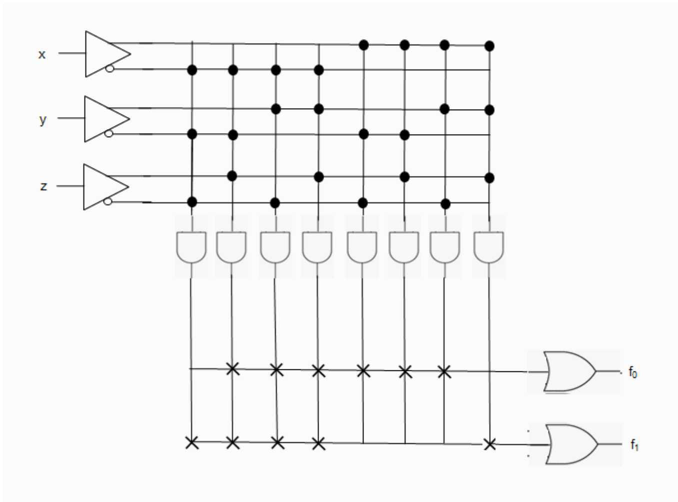
f= A+B+C+D= w'y'z+w'xy+wyz+wxy'

4. $f_0(x,y,z) = xy'+yz'+zx'$
$f_1(x,y,z) = x'+xyz$

(a) Realize f_0 and f_1 using a programmable read-only memory (PROM) with 3 inputs x,y,z and two outputs f_0 and f_1. Draw the final PROM logic diagram.

$f_0 = xy'(z+z')+(x+x')yz'+x'(y+y')z = x'y'z+x'yz'+x'yz+xy'z'+xy'z+xyz'$
$= \sum m(1,2,3,4,5,6)$

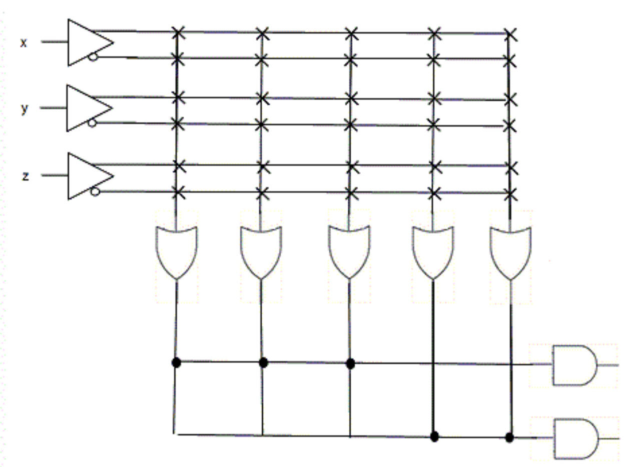
$f_1 = x'+xyz = x'(y+y')(z+z')+xyz = \sum m(0,1,2,3,7)$



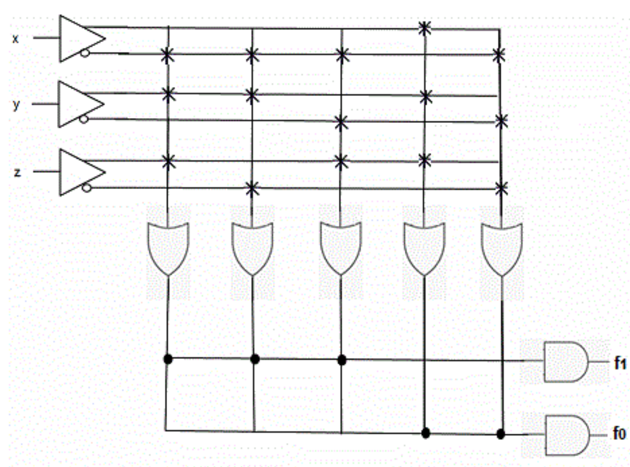
(b) Realize the same f_0 and f_1 using the PLD shown on the left below. This is a PLD similar to a programmable logic array (PLA), but with the AND and OR gate arrays switched. Using your knowledge of how PLD's work in general, complete the diagram on the right by labelling the outputs, and showing with X's which switches you would not blow. Show your work and explain your logic.

Since the inputs are first OR'd then AND'd we will have products of sums, ie CNF's. Now from part (a) above, f_0 will have 2 maxterms and f_1 4:

$f_0 = \Pi M(0,7) = (x+y+z)(x'+y'+z')$
$f_1 = \Pi M(4,5,6) = (x'+y+z)(x'+y+z')(x'+y'+z)$



PLD device



Your answer