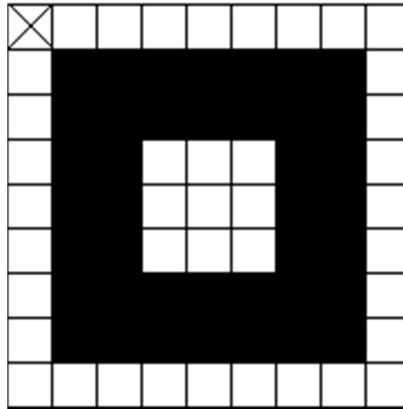


Final Exam Solutions

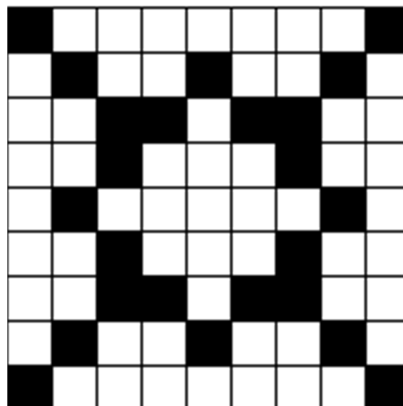
Instructions: Write your name above and on the top of the second page. Do not remove the staple. Write answers here or in the bluebook provided as directed. This exam will be collected along with the blue book. 75 minutes closed book/notes no electronic devices permitted.

1. Consider the blob X consisting of 80 object pixels arranged in a 9×9 square: $X = \{(0,0), \dots, (8,8)\}$, where the middle pixel of this square, the pixel at $(4,4)$, is a background pixel.

(a) Find $S_8(X)$, the maximum ball skeleton of X using 8-connected distance measure. Show your answer by darkening each skeleton pixel in $S_8(X)$ on the graph below. Explain your reasoning in the bluebook.



(b) Find $S_4(X)$, the maximum ball skeleton using the 4-connected distance measure. Show your answer by darkening each pixel in $S_4(X)$ on the graph below. Explain your reasoning in the bluebook.



The max balls for part a all have radius 1. For part b, the skeleton pixels on the outer ring (first/last row/col) all have radius 0, next ring in radius 1, next ring in radius 2.

2. The discrimination boundaries for a three-class decision problem over a two-dimensional feature space of vectors \mathbf{x} are shown below. They intersect at $\mathbf{x} = (1,1)$.

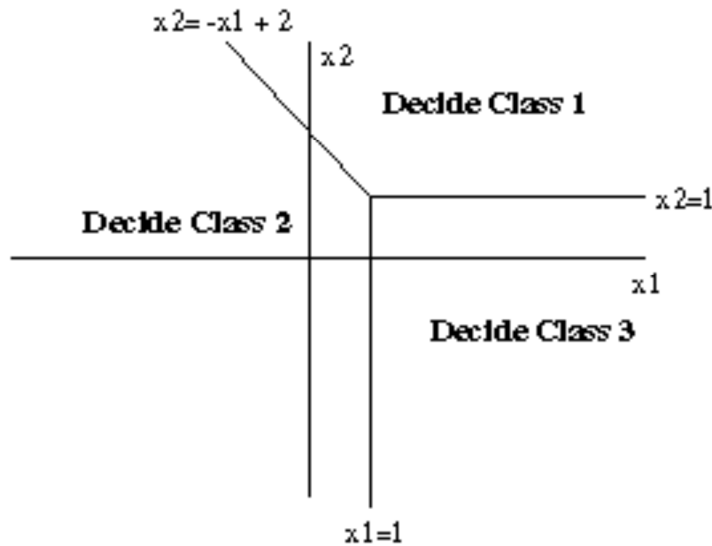
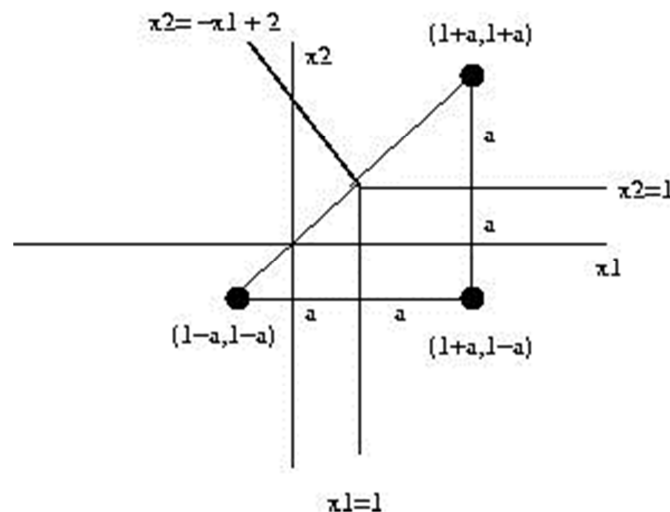


Figure 2

(a) Specify the (x_1, x_2) coordinates of 3 training atoms so that the minimum distance classifier trained using just this training data set agrees with the figure above.

For any $a > 0$, the three points on the diagram below constitute such a training set. The discrimination regions coincide with the given problem statement.



(b) Is the set of 3 training data answering (a) above unique? If not, specify all sets of 3 training data which solve (a) above.

No its not unique. The solution above works with any scalar $a > 0$.

(c) Could Fig 2 be consistent with a Gaussian minimum-error classifier? If yes, specify mean vectors and covariance matrices $\{\mu_i, \Sigma_i, i=1..3\}$, also prior probabilities $\{\pi_i, i=1..3\}$, which make them consistent. If no, explain why not.

Yes. Means equal to the three points in the above figure, variances all identity matrices, and equal priors.

3. In a certain image we have found $n \geq 3$ objects $R_1 \dots R_n$. We see that R_1 is blue, R_2 is green, R_3 is yellow and that R_i is adjacent to R_{i+1} for $i=1..n-1$. We wish to classify these objects using label set $\{O_1, \dots, O_m\}$. The rules are that only O_1 can be blue, and O_i and O_{i+1} must be adjacent for $i=1..m-1$. Now in the relaxation algorithm for constraint propagation we may check objects in any fixed order. Show that if we check objects in the order R_1 to R_n the algorithm converges much faster than if we check in the reverse order, R_n to R_1 . Specify all consistent label sets, and the number of iterations to convergence in each case.

3. Check in order R_1 - R_n : Checking properties, we find that R_1 can only be O_1 , and that all other objects can be any of O_1 - O_n . Checking relations, first we check if R_1 , which must be O_1 , is consistent with the relations, it is. Then we check if R_2 is consistent. Since R_1 is O_1 and O_1 must be adjacent to O_2 , then R_2 must be O_2 . Checking R_3 , since R_2 is O_2 and O_2 must be adjacent to O_3 , then R_3 must be O_3 . And so on. We converge in just a single iteration classifying $R_i = O_i$ for $i=1..n$.

Check in order R_n - R_1 : Checking properties, we find that R_1 can only be O_1 , and that all other objects can be any of O_1 - O_n . Checking relations, first we check if R_n , which can be any O_1 - O_n , is consistent with the relations. It is, and we cannot eliminate any R_n label since the adjacent object R_{n-1} can also be any O_1 - O_n . And so on down to R_2 , which must be O_2 , since R_1 is O_1 . Thus, after one iteration we have labelled 2 objects. Each iteration yields one more object label (the last one checked), that means $n-1$ iterations. It takes $n-1$ as many iterations checking in this order as in the order R_1 - R_n .