CSE 473/573 Spring 2010 March 2, 2010

Midterm Exam Solution Set

Instructions: Answer all questions in the bluebook provided. closed book, notes. No electronic devices allowed.

1. Let M be a 3x3 mask whose values are all -1 except the center pixel whose value is +8. The origin of the mask is its center pixel.

(a) Find the 6x6 output when M is applied to the 6x6 image shown below. Note: some of the values in the output will be negative.

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	1	0	0	0
0	1	1	0	0	0
0	0	0	0	0	0

Original Image

0	0	0	-1	-1	-1
0	0	0	-1	8	-1
-1	-2	-2	-2	-1	-1
-2	5	5	-2	0	0
-2	5	5	-2	0	0
-1	-2	-2	-1	0	0

After Filtering With M

(b) Suppose we apply M to a binary image, then we threshold the output so that only pixels +8 or higher are marked as foreground pixels. What image features in the original binary image are detected?

The only foreground pixels in the result will be those which were foreground pixels in the original image which are completely surrounded by background, ie. isolated foreground pixels.

(c) Repeat (b) in the case where we change the threshold from +8 to +3. What image features in the original binary image are detected?

Foreground pixels in the original image that have at least 3 8-connected background pixels. These correspond to most inner boundary pixels, for instance those along straight bounding edges of blobs and bounding edges of holes in blobs. Pixels in the interior of blobs surrounded by other blob pixels will not be detected, nor will pixels which are not themselves foreground pixels in the original image.

2. Let X be an 8x8 binary image which contains exactly 23 foreground pixels.

(a) How many nodes are there in the T-Pyramid for X?

64+16+4+1=85

(b) What is the <u>minimum</u> number of <u>inner boundary pixels</u> in any 8x8 image which contains exactly 23 foreground pixels? Use 8-connectivity to define adjacency between foreground and background pixels. Justify your answer.

For minimum number, pack all 23 into one blob with no holes. Since 8-connectivity defines foreground-background adjacency, must have the inner boundary 4-connected. Thus must pack the 23 pixels as close as possible into a square. This works out to a 5x5 square with 2 of its corner pixels missing. The resulting (minimum) number of inner boundary pixels is 16.

(c) What is the <u>maximum</u> number of nodes in the <u>quadtree</u> for any 8x8 image with exactly 23 foreground pixels? Show how the 23 foreground pixels can be distributed so that the quadtree has this maximum number of nodes.

Must have all quadrants, and sub-quadrants of quadrants, occupied. So divide the 8x8 image into 16 2x2 squares and make one of the 4 squares in each a foreground pixel. Scatter the remaining 7 pixels among the 2x2 squares, but no more than 3 foreground pixels per 2x2 square. Then the quadtree will have exactly as many nodes as the T-Pyramid, 85. This is the "worst-case" image for quadtree representation in that no data compression relative to T-Pyramid can be achieved because no path through the quadtree terminates early.

3. Define the <u>limit of an image X</u> as follows: given an image X, pick an operator Ω, where Ω is one of the four operators dilation, erosion, opening, or closing. Next pick a structuring element B. Then Y is the limit of X using Ω and B if Y is the result of applying Ω using structuring element B to the image X infinitely often:

$$Y = X \Omega B \Omega B \Omega B \Omega B \ldots$$

(a) Specify Ω_1 and B_1 so that Y_1 is the limit of X using Ω_1 and B_1.

Closing, B={(0,0),(1,1),(2,2)}

(b) Specify Ω_2 and B_2 so that Y_2 is the limit of X using Ω_2 and B_2.

Erosion, B=any structuring element with 2 or more pixels

(c) Specify Ω_3 and B_3 so that Y_3 is the limit of X using Ω_3 and B_3.

There is none. Can't be erosion or opening since more pixels in Y. Can't be dilation since for any B, the extra pixels towards the bottom right would imply extra pixels in the top right. Can't be closing since no pairs of blobs have been closed.

(d) Specify Ω_4 and B_4 so that Y_4 is the limit of X using Ω_4 and B_4.

There is none. It is an opening using B={(0,0),(1,0),(0,1)} shifted one column to the left. But there is no single operation that produces the given Y_4.

(e) Specify Ω_5 and B_5 so that Y_5 is the limit of X using Ω_5 and B_5.

Dilation, B is a 3x3 square of foreground pixels with the origin in the center pixel. There are many other solutions B as well, many with less than 9 pixels. They all get full credit.