

Midterm Exam With Solutions

1. In the 8x8 image X shown below, all pixels shown with an * have gray level 41, all the other pixels have gray level 14.

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	*				*		
	*	*		*	*		
	*	*	*	*	*		
		*	*	*			
		*	*	*			
			*				
			*				

(a) Suppose we linearly smooth X using a 1-row by 3-column averaging mask whose origin is the center of the mask. How many of the 64 pixels in X will change value? What is the largest increase in brightness any pixel in the given X will have?

All pixels except those in the center of a run of 3 equal horizontal pixels will change. The pixels changing include all pixels in the first and last columns (due to zero-bordering). Counting, there are only 15 pixels that will not change, so 49 will change. The max gray level increase is from 14 to $(1/3)*14+(2/3)*41 = 32$, so largest gray level increase is 32-14=18 (at pixel (3,4) using matrix notation).

(b) Suppose instead we smooth X using a median filter with the same mask size and origin as in (a). How many of the 64 pixels in X will change value? What is the largest increase in brightness any single pixel in X will have?

First zero border, and then count the pixels which are equal to the median of the set of three horizontal pixels with itself as the pixel in the center of the three. There are 6 such in the first and 2nd rows, 7 in the 3rd, 7th and 8th rows, and 8 in the 4th-6th rows for a total of 57 that don't change. That leaves 7 that change. The largest gray level increase is 41-14=27 (again at pixel (3,4)).

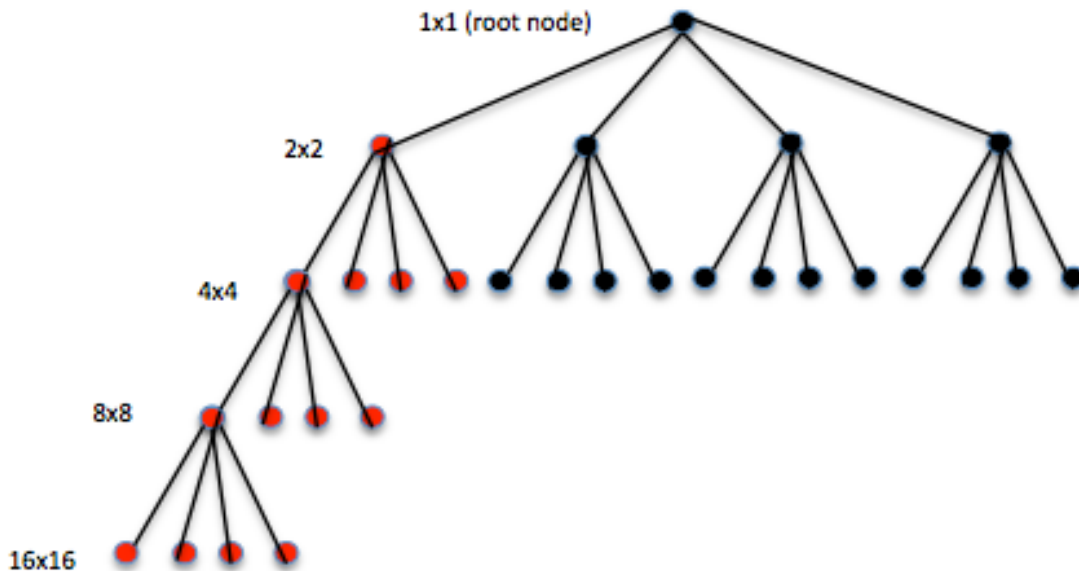
2. Consider a binary 16x16 image Y which contains two foreground blobs, one is of size 2x2=4 pixels and the other of size 1 pixel. Assume that this pair of foreground blobs are not connected to each other (the blobs are neither 4-connected nor 8-connected to each other).

(a) How many nodes will there be in the T-pyramid for Y?

$$16 \times 16 + 8 \times 8 + 4 \times 4 + 2 \times 2 + 1 \times 1 = \underline{341 \text{ nodes}}$$

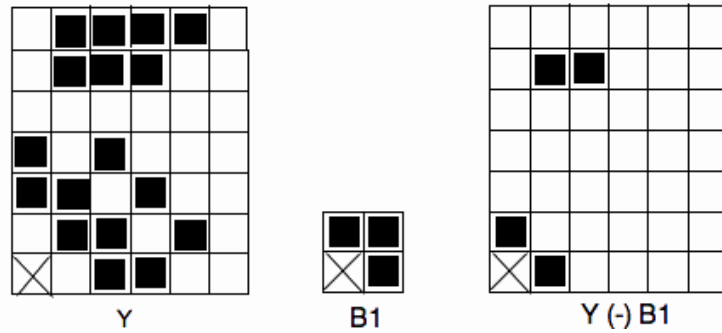
(b) What is the maximum, and the minimum, number of nodes in the quadtree for Y? For best part credit, show where you would place the blobs in Y and explain your reasoning.

Maximum: set the 2x2 in the center of Y, forcing 4 paths in the quadtree to descend down to the pixel (16x16) level. Then set the 1x1 on any corner of any of the major quadrants. Get 13 nodes each quadrant due to the 2x2 (nodes in red below) plus 8 more due to the 1x1 (red nodes at 8x8 and 16x16 levels), plus the root node, for total of $4 \times (13) + 8 + 1 = \underline{61 \text{ nodes max.}}$

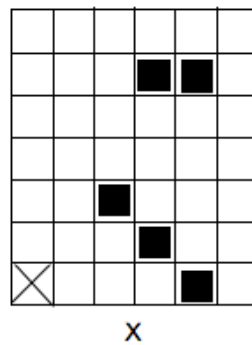


Minimum: set the 2x2 in a corner of any quadrant with the 1x1 one pixel separated from it by one pixel in the same quadrant. Then the quadtree contains one path to the 16x16 level (red nodes above), paths to the three other 2x2 nodes, and the root node, a total of 17 nodes min.

3. (a) Y is a binary image with foreground pixels shown in black. Find the erosion of Y by the structuring element $B_1 = \{(1,0), (0,1), (1,1)\}$. Give your answer graphically, with foreground pixels shown in black and the origin marked as above.



(b) Find the structuring element B_2 so that Y above is the dilation of the binary image X below. Give your answer graphically, with foreground pixels shown in black and the origin marked.



First note that $(4,5)$ is in X but not Y , so the origin is not a foreground pixel in B . Now put the origin of B at $(4,5)$. B must be some subset of the 7 black pixels in the first two rows of Y . But the left 2 can't be in B , else when we moved the origin to the pixel at $(3,5)$ we would get additional pixels in Y . Also the pixel above $(4,5)$ and the two to its left must be in B , else these pixels would not appear in Y . So far, it seems there are two possible B 's:



But when the origin of B is put on some of the other pixels, for instance $(4,0)$, we see the one on the left is the solution, the other does not work. For instance, using the one on the right, $(3,1)$ in Y should be black but it is not.