

Final Exam Solution Set

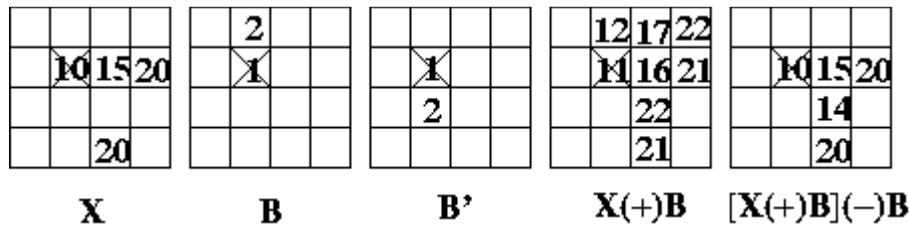
Instructions: Answer all questions in the bluebook provided. 75 min, open book/notes.

1. (a) Let X be any grey-scale image with $n > 0$ nonzero pixels, and B any grey-scale structuring element. Is it possible that the erosion of X by B contains more than n nonzero pixels? Justify your answer carefully.

(b) Determine the grey-scale closing of X' by B' (see figure below). Show your work. You may give your answer either as a figure or as a pointset from $Z \times Z \times G$.

(a) No. A necessary condition for a pixel to survive erosion is that all pixels in B must align with pixels in X (the surviving pixel will be at the origin of B). Now select any single pixel in B and line it up with the first pixel in X , then the second, and so on to the n th. Each such alignment can produce at most one pixel in the erosion.

(b) Closing=erosion of the dilation. For dilation, flip B to get B' . Answer is the rightmost figure.

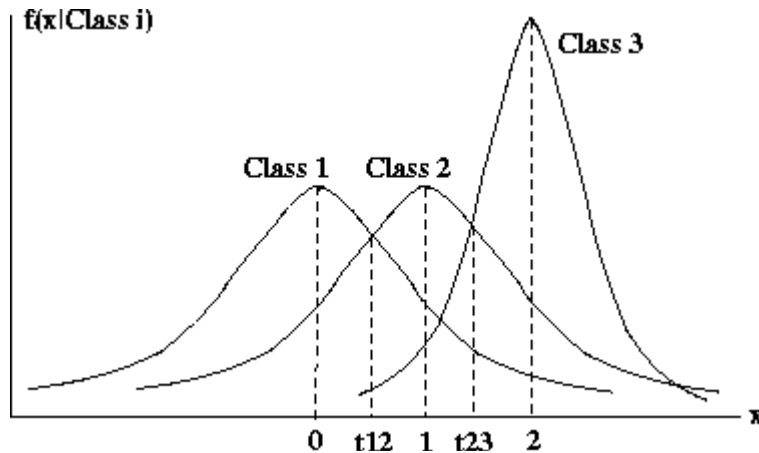


2. We are given a single scalar feature x to classify an object into one of 3 possible classes. The class-conditional probabilities of x are Gaussian with mean 0 variance 1 for Class 1, Gaussian with mean 1 variance 1 for Class 2, and Gaussian with mean 2 variance 1/4 for Class 3. Each classes has 1/3 a priori probability.

(a) What is the minimum error decision rule? In other words, specify which class you will guess as a function of x .

(b) Describe a specific application for which the minimum error decision rule found in part (a) above would not be a good rule to follow. Make it clear why the minimum error rule would not work well in the application you describe.

(a) Minimum error decision rule is to decide Class i when the class-conditional density for Class i is greatest among the 3 classes.



So our decision rule is: decide Class 1 when $x < t_{12}$, Class 2 when $t_{12} \leq x \leq t_{23}$ and Class 3 when $x > t_{23}$.

Now $t_{12} = 1/2$ by symmetry. To determine t_{23} , we must equate the densities for Class 2 and Class 3. After taking natural logs and cancelling common terms,

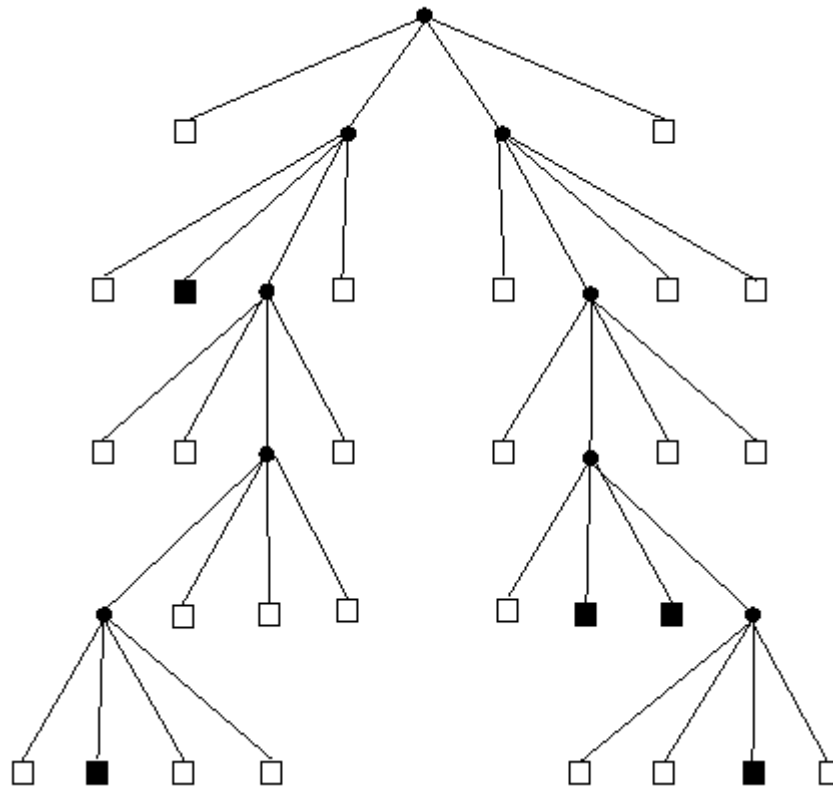
$$-\frac{1}{2}(x - 1)^2 = \ln 2 - 2(x - 2)^2$$

Expanding and using the quadratic formula, there are two roots. Referring to the figure, we need the lesser of the two for t_{23} . This is

$$t_{23} = \frac{7 - \sqrt{4 + \ln 2}}{3} = 1.61$$

(b) A minimum error classifier will work badly in any circumstance in which the cost of different kinds of errors is very different. For instance, if the cost of deciding Class 3 when the true class is Class 2 is very high, we would need to shift t_{23} to the right to minimize the expected cost. Any such application would require we use Bayesian rather than minimum error classification.

3. *The quadtree representation of an image is given in the figure below. Reading left to right on any level of the tree, the nodes represent the NW, NE, SE and SW quadrants in that order. Leafs shown as open squares represent background regions of the image, those leafs shown as closed squares are foreground regions. Foreground pixels have value 1, background pixels have value 0. The lowest level of the tree represents single pixels.*



(a) Determine the total area (in pixels) of all foreground regions in this image.

(b) Compute the non-central moments m_{00} and m_{01} of this image. Assume that $(i,j)=(0,0)$ is the upper left hand pixel in the image, with i the row index increasing downward, and j the column index increasing rightward. Show your work.

(a) Each black square on the lowest level is 1×1 foreground pixel, each on the next level is $2 \times 2 = 4$ pixels, each two levels up from that is 8×8 pixels. So the total area is $2 \times 1 + 2 \times 4 + 1 \times 64 = 74$ pixels.

(b) m_{00} is the same as the area, so it equals 74 as above. To get m_{01} , compute

$$m_{01} = \sum j$$

where the sum is taken over all foreground pixels and j is the column number.

Reconstructing the image from the quadtree, it looks like the figure shown below. There are 8 pixels in cols 24-28, 10 in col 29, and 12 in cols 30 and 31. So adding as above,

$$m_{01} = 8 \times (24 + 25 + 26 + 27 + 28) + 10 \times (29) + 12 \times (30 + 31) = 2,062$$

