

Final Exam

Instructions: Show all work and answers in the bluebook provided, except where specifically directed to put your answer on this exam sheet. 75 min, closed book/notes.

1. (a) Shown below in Fig. 1a is a gray level image derived from the painting "Broadway Boogie Woogie" by Piet Mondrian. In Figs. 1b-1e are shown the gray level dilation, erosion, opening and closing of Fig. 1a. Decide which figure corresponds to which gray level morphology operation. Justify your choices by mentioned what features of each of the images led you to make the choices you did.

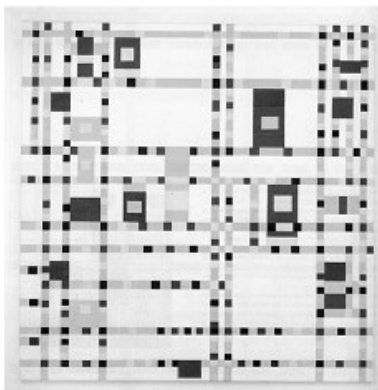


Figure 1a



Figure 1b



Figure 1c

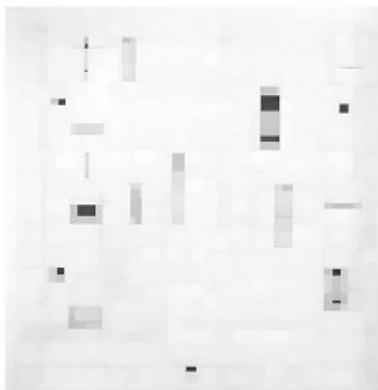


Figure 1d

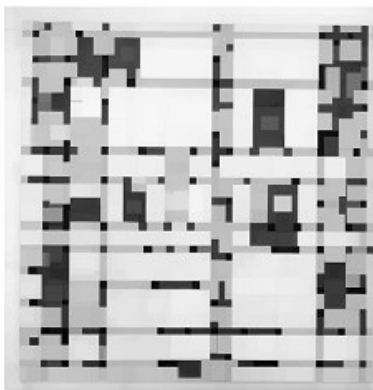
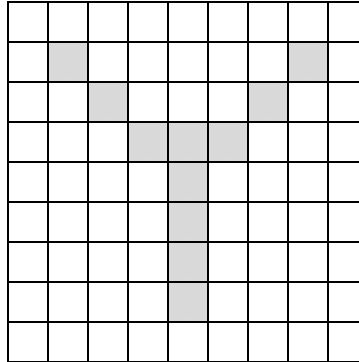


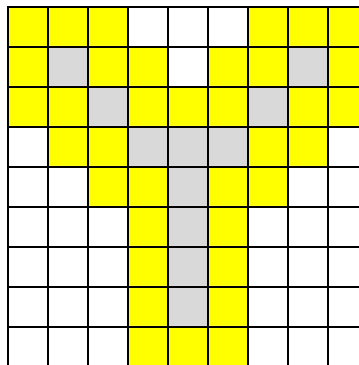
Figure 1e

Fig. 1b is darker so it's either erosion or opening. The lines are thicker, so it's erosion.
 Fig. 1d is lighter so it's either dilation or closing. Dark blobs are smaller, so it's dilation.
 Fig. 1c is lighter so by elimination it's closing, similarly Fig 1e is opening.

(b Shown below is the maximum ball skeleton constructed using the chessboard (8-adjacency) metric for a blob B. The skeleton pixels are shaded gray. What is the area of the smallest blob B that has this skeleton and contains at least one pixel which is not a skeleton pixel? Justify your answer.



All these skeleton pixels must have the same quench function value, otherwise the ball of the larger one would contain the ball of the smaller adjacent one so the smaller would not be a skeleton pixel. Setting the quench function value to be one for each skeleton pixel, the total number of pixels in B is 47.



2. (a) $f(i,j)$ shown below is a gray level image. The gray level value of each non-zero pixel are shown, e.g. $f(-2,-3)=1$, and for each blank pixel $f(i,j)=0$. The origin $(i,j)=(0,0)$ is the pixel $f(0,0)=3$ and the i and j axes are shown. Complete the table by computing the indicated moments, writing your answers directly in the table on this sheet. Show your work in the bluebook.

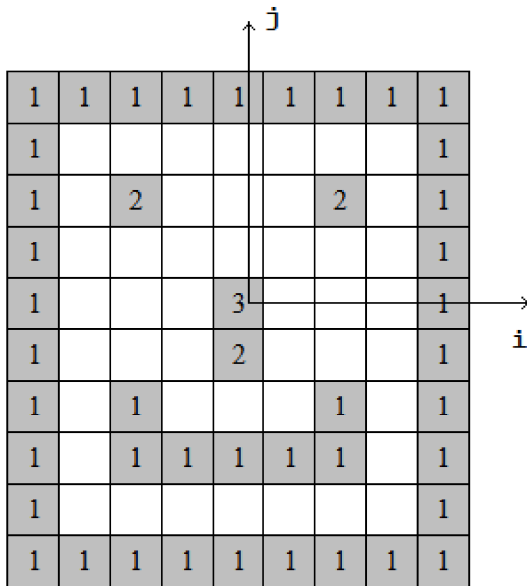


Image $f(i, j)$

ordinary moments		central moments	
m_{00}	50	μ_{00}	50
m_{01}	-29	μ_{01}	0
m_{10}	0	μ_{10}	0

Complete this table

$$m_{00} = \sum \sum f(i,j) = 41 \cdot 1 + 3 \cdot 2 + 1 \cdot 3 = 50$$

$$m_{01} = \sum \sum j f(i,j) = 9 \cdot 4 + 2 \cdot 3 + 6 \cdot 2 + 2 \cdot 1 + 5 \cdot 0 - 4 \cdot 1 - 4 \cdot 2 - 7 \cdot 3 - 2 \cdot 4 - 9 \cdot 5$$

$$= 36 + 6 + 12 + 2 - [4 + 8 + 21 + 8 + 45]$$

$$= 56 - 86 = -30$$

$$m_{10} = \sum \sum i f(i,j) = 0 \text{ since for all } i \neq 0, f(i,j) = f(-i,j) \text{ and thus all terms cancel out.}$$

$$\mu_{00} = m_{00} = 50 \text{ (}\mu_{00} = m_{00} \text{ always true)}$$

$$\mu_{01} = \mu_{10} = 0 \text{ (always true)}$$

(b) The image $f(i,j)$ above has symmetry about the j -axis, i.e. $f(i,j) = f(-i,j)$. What set of ordinary moments m_{pq} are guaranteed to be zero for any figure with symmetry around the j -axis? Specify your answer as a function of p and q . Consider all moments (p, q non-negative integers).

We need $(i)^p (j)^q = -(-i)^p (j)^q$ for all i and j . This will be true for all pairs of non-negative integers (p, q) such that p is odd.

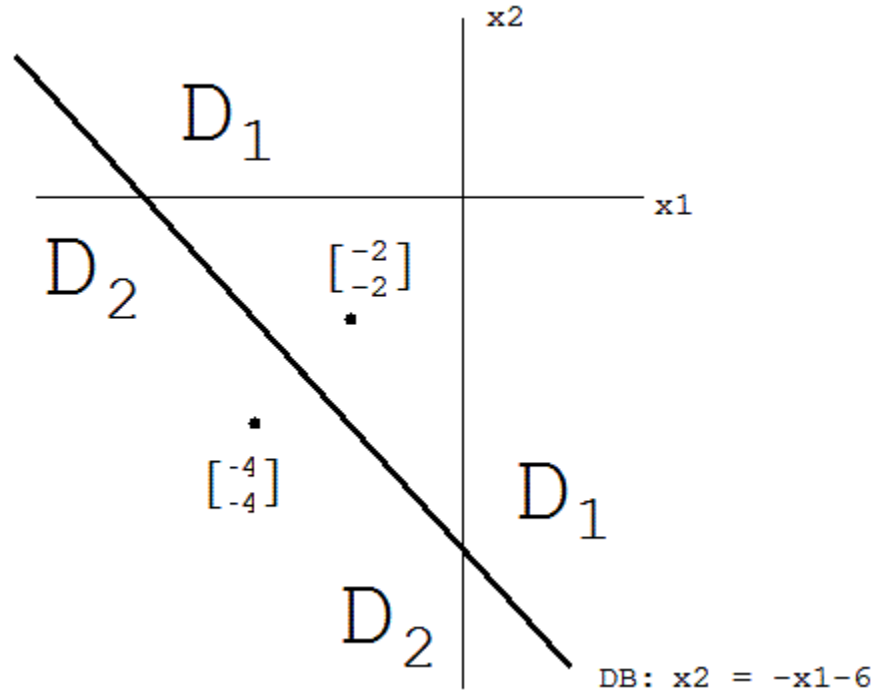
(c) If the gray levels of each pixel in an image $g(i,j)$ were doubled, what would be effect be on each of the central moments μ_{pq} ? Specify your answer as a function of p and q . Consider all moments (p, q non-negative integers).

All central moments are linear in the gray levels $f(i,j)$. So doubling all gray levels will result in doubling of each of the central moments (also the ordinary moments).

3. Consider a 2-class object recognition problem $\Omega = \{\omega_1, \omega_2\}$ with a 2-D feature space. The two classes have Gaussian density functions and have equal prior probabilities. Their covariance matrices are $\Sigma_1 = \Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and their means are

$$\mu_1 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}, \mu_2 = \begin{bmatrix} -4 \\ -4 \end{bmatrix}$$

(a) Sketch the minimum-error decision boundary, labeling the decision regions and giving a formula for your decision boundary.



(b) If the prior probability for class 1 was increased and all else remain as in (a), what direction would the decision boundary move, and how would its shape change?

DB moves down and to the left, shape does not change.

(c) If the covariance matrix for class 1 was scaled larger and all else remain as in (a), what direction would the decision boundary move, and how would its shape change?

DB moves down and to the left, shape becomes parabolic.

(d) Specify a training data set so that the nearest-neighbor decision boundary is exactly the same as the minimum-error decision boundary in (a) above.

Simplest is a training dataset with atoms at the two means: $\left\{ \left(\begin{bmatrix} -2 \\ -2 \end{bmatrix}, 1 \right), \left(\begin{bmatrix} -4 \\ -4 \end{bmatrix}, 2 \right) \right\}$.