1.

```
function en = EN12(InIm)
    InIm = double(InIm);
    [nr nc] = size(InIm);

    pad = 1;
    PImg = double(ones(nr+(pad*2),nc+(pad*2)));

    for i=(1+pad):(nr+pad) % padding background(ones)
        for j=(1+pad):(nc+pad)
            PImg(i,j) = InIm(i-pad,j-pad);
        end
    end

    [LabeledImage1 noOfBlobs1] = MyBwLabel(PImg);
    S = noOfBlobs1;
    [LabeledImage2 noOfBlobs2] = MyBwLabel(~PImg);
    N = noOfBlobs2 - 1;

    E = S - N;
    en = E;
end

function [img noOfBlobs] = MyBwLabel(InIm)
% foreground pixels are 0's
[nr nc] = size(InIm);
img = double (InIm);
blobno = 1;
for r=1:nr
    for c=1:nc
        if(img(r,c)==0)
            blobno = blobno + 1;
            img = CreateBlob12(img,blobno,r,c);
        end
    end
end
noOfBlobs = max(max(img))-1;
end

% Creates a matrix of single blob
function blobmatrix = CreateBlob12(image,blobno,x,y)
mover = [-1 -1 +0 +1 +1 +1 +0 -1 -2 +0 +2 +0];
movec = [+0 +1 +1 +1 +0 -1 -1 -1 +0 +2 +0 -2];
[nr nc] = size(image);
visitedcells(1,1) = x;
visitedcells(1,2) = y;
count = 1;
visited = 1;
image(x,y) = blobno;
while((visited<=count) || (count==visited==1))
    r = visitedcells(visited,1);
    c = visitedcells(visited,2);
    image(r,c) = blobno;
    for i=1:12
```

```
        if(r+mover(i)>=1 && r+mover(i)<=nr && c+movec(i)>=1 && c+movec(i)<=nc )
            if(image(r+mover(i),c+movec(i))==0)
                count = count + 1;
                visitedcells(count,1) = r+mover(i);
                visitedcells(count,2) = c+movec(i);
                image(r+mover(i),c+movec(i)) = blobno;
            end
        end
    end
    visited = visited + 1;
end
blobmatrix = image;
end
```

2.

$$\mu_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mu_2 = \begin{bmatrix} 4 \\ -4 \end{bmatrix} \Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mu_3 = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \Sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$det(\Sigma_1) = det(\Sigma_2) = det(\Sigma_3) = 1$$

$$\Sigma_1^{-1} = \Sigma_2^{-1} = \Sigma_3^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\log p(x|\omega_1) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log(det(\Sigma_1)) - \frac{1}{2}[(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1)]$$

$$\log p(x|\omega_2) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log(det(\Sigma_2)) - \frac{1}{2}[(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2)]$$

$$\log p(x|\omega_3) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log(det(\Sigma_3)) - \frac{1}{2}[(x-\mu_3)^T\Sigma_3^{-1}(x-\mu_3)]$$

$$p(\omega_1) = p(\omega_2) = p(\omega_3) = \frac{1}{3}$$

1.1. Decision boundary between class 1 and class 2

$$p(\omega_1|x) = p(\omega_2|x)$$

$$\frac{p(x|\omega_1)p(\omega_1)}{p(x)} = \frac{p(x|\omega_2)p(\omega_2)}{p(x)}$$

$$p(x|\omega_1) = p(x|\omega_2) \implies \log p(x|\omega_1) = \log p(x|\omega_2)$$

$$-\frac{1}{2}[(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1)] = -\frac{1}{2}[(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2)]$$

$$(x_1-2)^2 + (x_2-2)^2 = (x_1-4)^2 + (x_2+4)^2$$

$$\boxed{x_1 - 3x_2 - 6 = 0}$$

1.2. Decision boundary between class 1 and class 3

$$p(\omega_1|x) = p(\omega_3|x)$$
$$\frac{p(x|\omega_1)p(\omega_1)}{p(x)} = \frac{p(x|\omega_3)p(\omega_3)}{p(x)}$$
$$p(x|\omega_1) = p(x|\omega_3) \implies \log p(x|\omega_1) = \log p(x|\omega_3)$$
$$-\frac{1}{2}[(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1)] = -\frac{1}{2}[(x-\mu_3)^T\Sigma_3^{-1}(x-\mu_3)]$$
$$(x_1-2)^2 + (x_2-2)^2 = (x_1+2)^2 + (x_2+2)^2$$
$$\boxed{x_1 + x_2 = 0}$$

1.3. Decision boundary between class 2 and class 3

$$p(\omega_2|x) = p(\omega_3|x)$$
$$\frac{p(x|\omega_2)p(\omega_2)}{p(x)} = \frac{p(x|\omega_3)p(\omega_3)}{p(x)}$$
$$p(x|\omega_2) = p(x|\omega_3) \implies \log p(x|\omega_2) = \log p(x|\omega_3)$$
$$-\frac{1}{2}[(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2)] = -\frac{1}{2}[(x-\mu_3)^T\Sigma_3^{-1}(x-\mu_3)]$$
$$(x_1-4)^2 + (x_2+4)^2 = (x_1+2)^2 + (x_2+2)^2$$
$$\boxed{-3x_1 + x_2 + 6 = 0}$$

2. Problem 2 b

$$\mu_1 = \begin{bmatrix}2\\2\end{bmatrix} \Sigma_1 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\mu_2 = \begin{bmatrix}4\\-4\end{bmatrix} \Sigma_2 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\mu_3 = \begin{bmatrix}-2\\-2\end{bmatrix} \Sigma_3 = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$$

$$det(\Sigma_1) = det(\Sigma_2) = 1$$
$$det(\Sigma_3) = 4$$
$$\Sigma_1^{-1} = \Sigma_2^{-1} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$
$$\Sigma_3^{-1} = \begin{bmatrix}0.5 & 0\\0 & 0.5\end{bmatrix}$$

$$\log p(x|\omega_1) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log(det(\Sigma_1)) - \frac{1}{2}[(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1)]$$
$$\log p(x|\omega_2) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log(det(\Sigma_2)) - \frac{1}{2}[(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2)]$$
$$\log p(x|\omega_3) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log(det(\Sigma_3)) - \frac{1}{2}[(x-\mu_3)^T\Sigma_3^{-1}(x-\mu_3)]$$
$$p(\omega_1) = p(\omega_2) = p(\omega_3) = \frac{1}{3}$$

2.1. Decision boundary between class 1 and class 2

$$p(\omega_1|x) = p(\omega_2|x)$$
$$\frac{p(x|\omega_1)p(\omega_1)}{p(x)} = \frac{p(x|\omega_2)p(\omega_2)}{p(x)}$$
$$p(x|\omega_1) = p(x|\omega_2) \implies \log p(x|\omega_1) = \log p(x|\omega_2)$$
$$-\frac{1}{2}[(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1)] = -\frac{1}{2}[(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2)]$$
$$(x_1-2)^2 + (x_2-2)^2 = (x_1-4)^2 + (x_2+4)^2$$
$$\boxed{x_1 - 3x_2 - 6 = 0}$$

2.2. Decision boundary between class 1 and class 3

$$p(\omega_1|x) = p(\omega_3|x)$$
$$\frac{p(x|\omega_1)p(\omega_1)}{p(x)} = \frac{p(x|\omega_3)p(\omega_3)}{p(x)}$$
$$p(x|\omega_1) = p(x|\omega_3) \implies \log p(x|\omega_1) = \log p(x|\omega_3)$$
$$-\frac{1}{2}\log(det(\Sigma_1)) - \frac{1}{2}[(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1)] = -\frac{1}{2}\log(det(\Sigma_3)) - \frac{1}{2}[(x-\mu_3)^T\Sigma_3^{-1}(x-\mu_3)]$$
$$-\frac{1}{2}[(x_1-2)^2 + (x_2-2)^2] = -\frac{1}{2}\log(4) - \frac{1}{2}[\frac{(x_1+2)^2}{2} + \frac{(x_2+2)^2}{2}]$$
$$\boxed{x_1^2 + x_2^2 - 12x_1 - 12x_2 + 8 - \log(4) = 0}$$

2.3. Decision boundary between class 2 and class 3

$$p(\omega_2|x) = p(\omega_3|x)$$
$$\frac{p(x|\omega_2)p(\omega_2)}{p(x)} = \frac{p(x|\omega_3)p(\omega_3)}{p(x)}$$
$$p(x|\omega_2) = p(x|\omega_3) \implies \log p(x|\omega_2) = \log p(x|\omega_3)$$
$$-\frac{1}{2}\log(det(\Sigma_2)) - \frac{1}{2}[(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2)] = -\frac{1}{2}\log(det(\Sigma_3)) - \frac{1}{2}[(x-\mu_3)^T\Sigma_3^{-1}(x-\mu_3)]$$
$$-\frac{1}{2}[(x_1-4)^2 + (x_2+4)^2] = -\frac{1}{2}\log(4) - \frac{1}{2}[\frac{(x_1+2)^2}{2} + \frac{(x_2+2)^2}{2}]$$
$$\boxed{x_1^2 + x_2^2 - 20x_1 - 12x_2 + 56 - \log(4) = 0}$$

3. Problem 3

$$\mu_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mu_3 = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \Sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$det(\Sigma_1) = det(\Sigma_2) = det(\Sigma_3) = 1$$
$$\Sigma_1^{-1} = \Sigma_2^{-1} = \Sigma_3^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\log p(x|\omega_1) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log(det(\Sigma_1)) - \frac{1}{2}[(x-\mu_1)^T \Sigma_1^{-1}(x-\mu_1)]$$
$$\log p(x|\omega_2) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log(det(\Sigma_2)) - \frac{1}{2}[(x-\mu_2)^T \Sigma_2^{-1}(x-\mu_2)]$$
$$\log p(x|\omega_3) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log(det(\Sigma_3)) - \frac{1}{2}[(x-\mu_3)^T \Sigma_3^{-1}(x-\mu_3)]$$
$$p(\omega_1) = p(\omega_2) = p(\omega_3) = \frac{1}{3}$$
$$\lambda(\omega_1|\omega_1) = \lambda(\omega_2|\omega_2) = \lambda(\omega_3|\omega_3) = 0$$
$$\lambda(\omega_2|\omega_1) = \lambda(\omega_3|\omega_1) = 1$$
$$\lambda(\omega_1|\omega_2) = \lambda(\omega_3|\omega_2) = \lambda(\omega_2|\omega_3) = \lambda(\omega_1|\omega_3) = 2$$

Expected loss for class 1 :

$$\begin{aligned} E[\lambda(\omega_1|\omega_s, x)] &= \lambda(\omega_1|\omega_1)P(\omega_1|x) + \lambda(\omega_1|\omega_2)P(\omega_2|x) + \lambda(\omega_1|\omega_3)P(\omega_3|x) \\ &= 2P(\omega_2|x) + 2P(\omega_3|x) \end{aligned}$$

Expected loss for class 2 :

$$\begin{aligned} E[\lambda(\omega_2|\omega_s, x)] &= \lambda(\omega_2|\omega_1)P(\omega_1|x) + \lambda(\omega_2|\omega_2)P(\omega_2|x) + \lambda(\omega_2|\omega_3)P(\omega_3|x) \\ &= P(\omega_1|x) + 2P(\omega_3|x) \end{aligned}$$

Expected loss for class 3 :

$$\begin{aligned} E[\lambda(\omega_3|\omega_s, x)] &= \lambda(\omega_3|\omega_1)P(\omega_1|x) + \lambda(\omega_3|\omega_2)P(\omega_2|x) + \lambda(\omega_3|\omega_3)P(\omega_3|x) \\ &= P(\omega_1|x) + 2P(\omega_2|x) \end{aligned}$$

3.1. Decision boundary between class 1 and class 2 is where the expected loss for two classes are equal,

$$E[\lambda(\omega_1|\omega_s, x)] = E[\lambda(\omega_2|\omega_s, x)]$$
$$2p(x|\omega_2) = p(x|\omega_1) \implies \log(2) + p(x|\omega_2) = \log p(x|\omega_1)$$
$$\log(2) + (x_1 - 4)^2 + (x_2 + 2)^2 = (x_1 - 2)^2 + (x_2 - 2)^2$$
$$\boxed{x_1 - 3x_2 - 6 - \frac{\log(2)}{4} = 0}$$

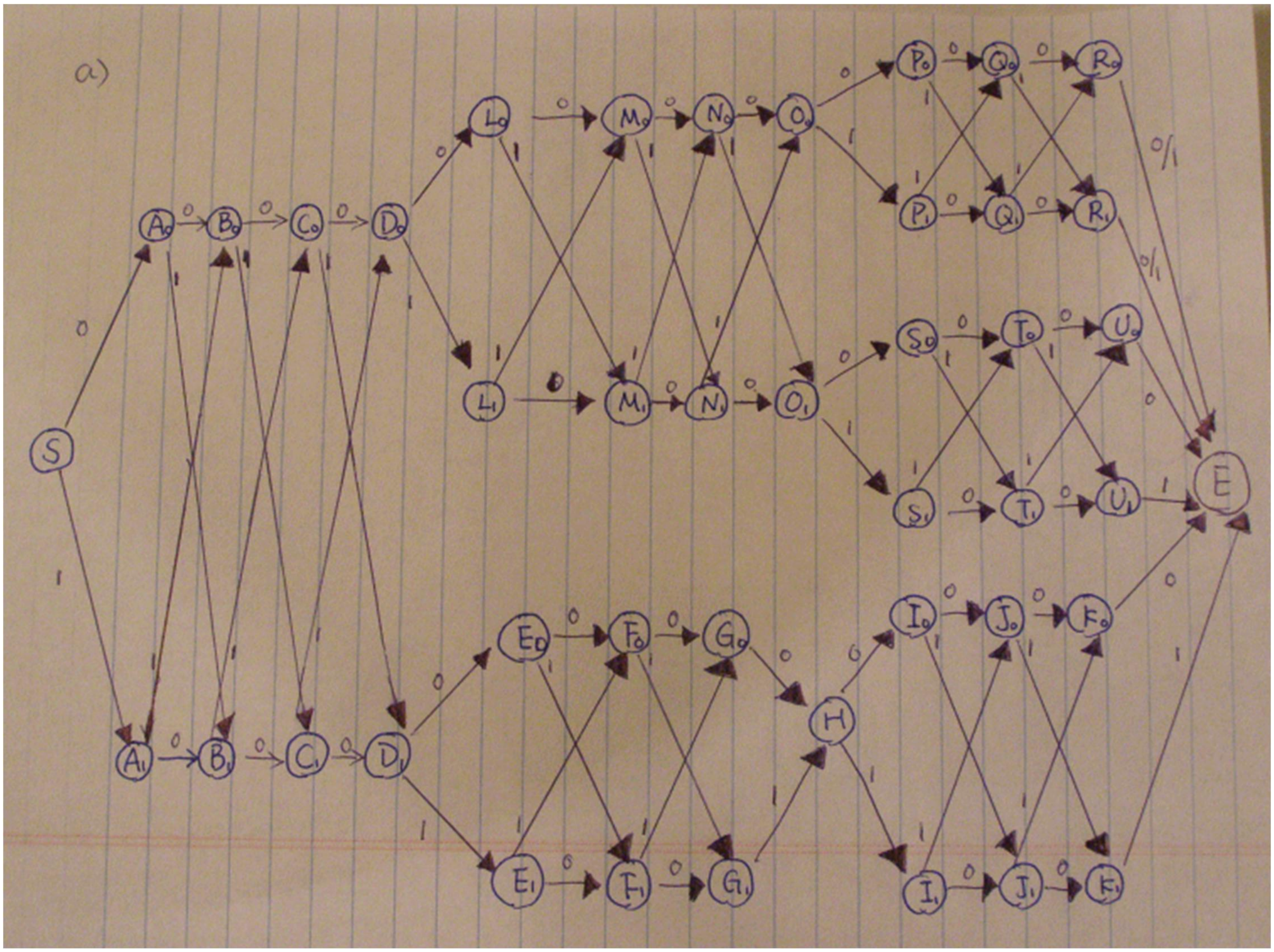
3.2. Decision boundary between class 1 and class 3 is where the expected loss for two classes are equal,

$$E[\lambda(\omega_1|\omega_s, x)] = E[\lambda(\omega_3|\omega_s, x)]$$
$$2p(x|\omega_3) = p(x|\omega_1) \implies \log(2) + p(x|\omega_3) = \log p(x|\omega_1)$$
$$\log(2) + (x_1 + 2)^2 + (x_2 + 2)^2 = (x_1 - 2)^2 + (x_2 - 2)^2$$
$$\boxed{x_1 + x_2 + \frac{\log(2)}{8} = 0}$$

3.3. Decision boundary between class 2 and class 3 is where the expected loss for two classes are equal,

$$E[\lambda(\omega_2|\omega_s, x)] = E[\lambda(\omega_3|\omega_s, x)]$$
$$p(x|\omega_3) = p(x|\omega_2) \implies \log p(x|\omega_3) = \log p(x|\omega_2)$$
$$(x_1 - 4)^2 + (x_2 + 4)^2 = (x_1 + 2)^2 + (x_2 + 2)^2$$
$$\boxed{-3x_1 + x_2 + 6 = 0}$$

4.



b) $V_n = \{S, A_0, A_1, B_0, B_1, C_0, C_1, D_0, D_1, E_0, E_1, F_0, F_1,$
$G_0, G_1, H, I_0, I_1, J_0, J_1, K_0, K_1, L_0, L_1, M_0, M_1,$
$N_0, N_1, O_0, O_1, P_0, P_1, Q_0, Q_1, R_0, R_1, S_0, S_1, T_0,$
$T_1, U_0, U_1\}$

$V_t = \{0, 1\}$

$P = \{S \to 0A_0 \mid 1A_1,\ A_0 \to 0B_0 \mid 1B_1,\ A_1 \to 1B_0 \mid 0B_1,$
$B_0 \to 0C_0 \mid 1C_1,\ B_1 \to 0C_1 \mid 1C_0,\ C_0 \to 0D_0 \mid 1D_1,$
$C_1 \to 1D_0 \mid 0D_1,\ D_0 \to 0L_0 \mid 1L_1,\ D_1 \to 0E_0 \mid 1E_1,$
$E_0 \to 0F_0 \mid 1F_1,\ E_1 \to 0F_1 \mid 1F_0,\ F_0 \to 0G_0 \mid 1G_1,$
$F_1 \to 0G_1 \mid 1G_0,\ G_0 \to 0H,\ G_1 \to 1H,$
$H \to 0I_0 \mid 1I_1,\ I_0 \to 0J_0 \mid 1J_1,\ I_1 \to 0J_1 \mid 1J_0$
$J_0 \to 0K_0 \mid 1K_1,\ J_1 \to 0K_1 \mid 1K_0,\ K_0 \to 0,$
$K_1 \to 1,\ L_0 \to 0M_0 \mid 1M_1,\ L_1 \to 0M_1 \mid 1M_0,$
$M_0 \to 0N_0 \mid 1N_1,\ M_1 \to 1N_0 \mid 0N_1,\ N_0 \to 0O_0 \mid 1O_1$
$N_1 \to 0O_1 \mid 1O_0,\ O_0 \to 0P_0 \mid 1P_1,\ O_1 \to 0S_0 \mid 1S_1,$
$P_0 \to 0Q_0 \mid 1Q_1,\ P_1 \to 0Q_1 \mid 1Q_0,\ Q_0 \to 0R_0 \mid 1R_1,$
$Q_1 \to 0R_1 \mid 1R_0,\ R_0 \to 0 \mid 1,\ R_1 \to 0 \mid 1$
$S_0 \to 0T_0 \mid 1T_1,\ S_1 \to 0T_1 \mid 1T_0,\ T_0 \to 0U_0 \mid 1U_1,$
$T_1 \to 1U_0 \mid 0U_1,\ U_0 \to 0,\ U_1 \to 1\}$