## Midterm Exam Solutions

Instructions: anwer all 3 questions in the bluebook provided. 50 min, open book, notes.

1. (a) Sketch a $4 \times 4$ binary image whose $T$-pyramid representation and quadtree representation are exactly the same. You do not have to give the representations, just the image.
(b) Sketch a $4 x 4$ binary image whose run-length code is the longest possible for any $4 \times 4$ binary image. You do not have to give the code, just the image.
(c) Sketch a 4x4 binary image containing a single 8-connected object whose chain code representation is much longer than the run-length code of the image. Only the image is required.

(a) and (b)

(c)

In (a), no branch of the quadtree can terminate early, so there is no compression relative to Tpyramid. In (b), there are two runs per row, the most possible for a $4 x 4$ image. In (c), there is only 1 run to be run-length coded, while 7 pixels must be included in the chain code, three of them included twice.
2.

| -1 | -1 | -1 |
| :---: | :---: | :---: |
| -1 | +1 | +1 |
| -1 | +1 | +1 |


| 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 5 | 5 | 5 | 0 |
| 0 | 5 | 5 | 5 | 0 |
| 0 | 5 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 |


| 20 | 10 | 0 | -10 | 0 |
| :---: | :---: | :---: | :---: | :---: |
| 10 | -5 | -10 | -15 | 0 |
| 0 | -10 | -10 | -10 | 0 |
| -10 | -15 | -10 | -5 | 0 |
| 0 | 0 | 0 | 0 | 0 |

M
X
(a) Consider a linear filter which uses the mask $\boldsymbol{M}$ shown above. What kind of features does this filter find: is it an edge-finder, a line-finder, or a corner-finder? Explain your choice.

When this mask is placed over an upper left hand corner of an object, it gives its maximum output. So it is a corner-finder. It cannot find all corners, just those aligned as upper left hand corners. It could also be considered a matched filter, one matched to a right angle corner on the upper left hand boundary of an object.
(b) Compute $\boldsymbol{Y}$, the output image when the image $\boldsymbol{X}$ is filtered by $\boldsymbol{M}$, and show your result by filling in the grey level values for $\boldsymbol{Y}$ in the above figure. What features has the filter "found?"
(c) Explain how you could use M, together with some other masks, to create a filter which would find not just features at one specific orientation, but at any orientation. Describe the set of masks you would use, and how you would use them. You need not give numerical values for all the masks you propose adding, just one or two of them.

The values of $\mathbf{Y}$ are shown. 20 is the largest. Note that the origin of $\mathbf{M}$ is on the outer boundary of the corner it is finding, not the inner boundary. So the 20 shows that we have found a corner with outer boundary pixel is at $(0,0)$. That is the only point of very strong output of this filter, thus the only corner successfully found. Note it is an upper left hand corner, as predicted in part (a).
(c) Explain how you could use M, together with other masks of your choice, to create a filter which would find not just features at one specific orientation, but at any orientation. Describe the set of masks you would use, and how you would use them. You need not give numerical values for all the masks you propose adding, just one or two.

To find upper right hand corners, the mask would be flipped about a vertical axis. For lower corners, these two masks would be flipped about a horizontal axis. Then for each pixel, all the masks are run, with the maximum output defined as the filter output. Beyond these four masks, others rotated 45 degrees could be used as well to find corners not aligned vertically and horizontally.
3. $B=\{(0,0)(1,0)(2,0)(0,-1)(0,-2)\}$
(a) Find $Y=B \oplus B$, the dilation of $B$ by $B$. Show your result as a graph, remembering to mark the cell which is the origin of $Y$ with an "ex."

(b) What is the smallest object $X$ that survives 3 erosions by B? Smallest means least number of object pixels, and survives means that after $X$ is eroded by $B$, and the resulting image eroded by $B$ again, and the resulting image eroded by $B$ again, the result still has at least one object pixel. Show your answer graphically. For best part credit, explain your reasoning step by step.

Start by asking what is the smallest object that survives one erosion. For a pixel to survive an erosion, all the pixels in the mask must also be object pixels when the object's origin is over that pixel. So the smallest object to survive one erosion is the mask itself.

So what is the smallest object to survive two erosions? After the first erosion, it must be the smallest object to survive one erosion, ie. as above, the mask itself. So start with a graph of the mask, then associate with each pixel in the mask all the pixels in a second copy of the mask when that second copy's origin is over that pixel. The result is the graph $B(+) B$ above.

Finally, repeat one more time. In other words, associate with each pixel B (+) B all the pixels in the mask when that pixel aligned with the origin of the mask. The result shown below, and is in fact B (+) B (+) B. This is the smallest object that survives 3 erosions with B.


Smallest object surviving 3 ecosions by B

