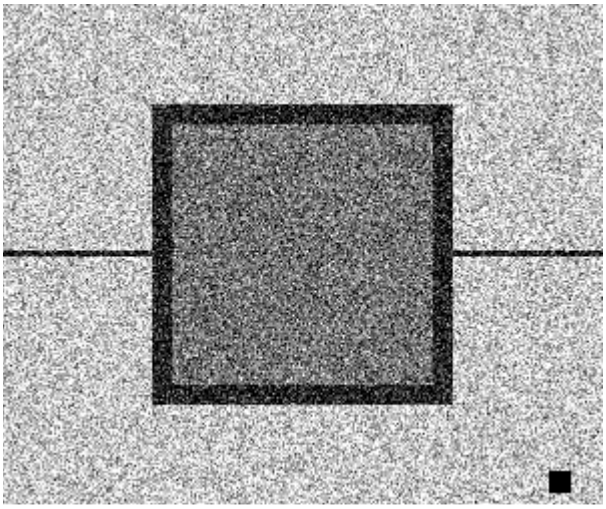
CSE 473/573 October 21 2008

Midterm Exam Solution Set

1. When the image on the left below was filtered using a smoothing filter, the result was the image on the right. The filter used was one of these: 1. Averaging filter; 2. rotating mask averaging filter; 3. Gaussian filter; 4. Median filter. The small black square on the lower right hand corner of the original image shows the size of the mask that was used, that small square is not part of the image.

(a) For each of the four possible filters, give at least one reason why you think it was, or was not, the filter actually used. Note that the mask is the same size as the thickness of the dark border around the square, and twice the thickness of the dark lines on either side of the square.

(b) If the same type of filter you selected in (a) was used but its mask reducted to only half the number of rows and and half the columns of the mask in (a), how would the appearance of the image after filtering be changed?



Original image. Note: the small black square on the lower right shows the size of the mask, it is not part of the image.

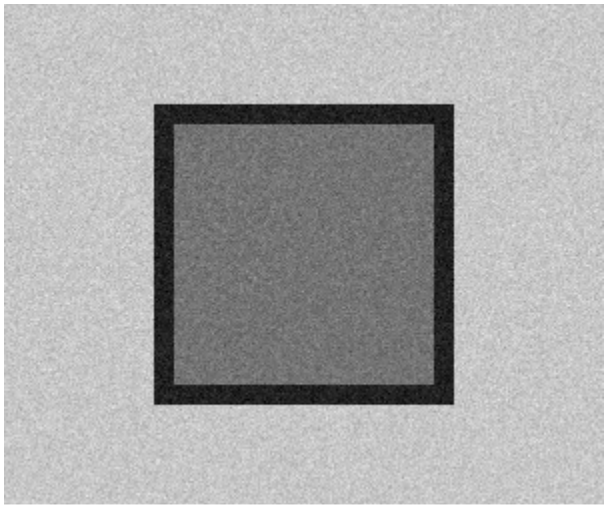
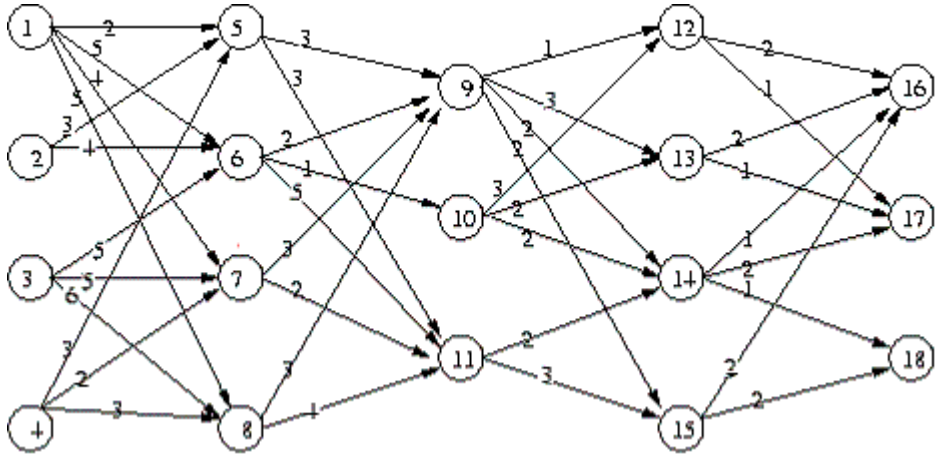


Image after filtering

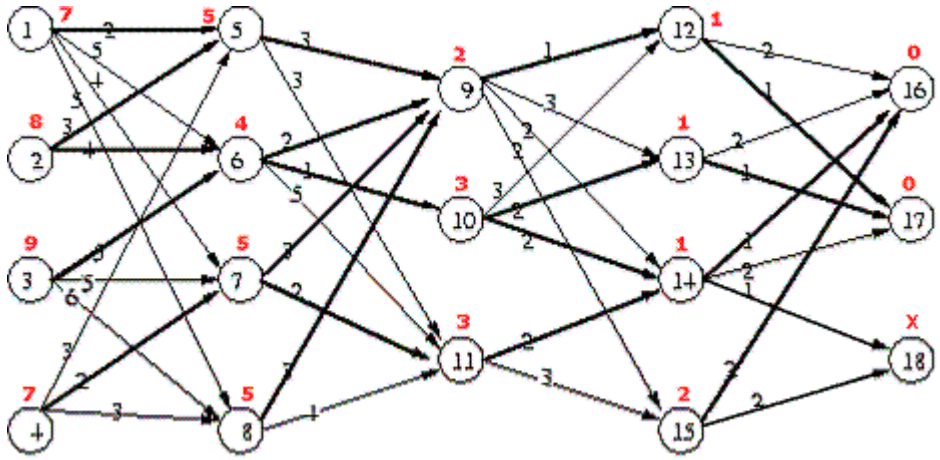
(a) Can't be the averaging filter since the edges are not blurred. Can't be the rotating mask averaging filter, since the thin black lines would be lighter but not disappear. Can't be the Gaussian filter because the edges are not blurred. Must be the median filter, since it reduces noise, eliminates small objects and does not blur edges.

(b) If the median filter mask were reduced as described, the two black lines outside the square would be clearly visible, along with the black border of the square. There would also be less smoothing in the rest of the image, since the median of a smaller number of pixels would be more variable as the window is moved around. So the remaining salt and pepper noise would be greater.

2. Find the minimum cost path using dynamic programming. Your path must start from one of nodes 1-4 and end at either node 16 or 17. If two or more paths are tied for the minimum, specify any one. Show your work.



The cost-to-go values are shown atop each node, and the optimal links shown as bold arrows for each node. Since nodes 1 and 4 both have the minimum cost of 7, there are minimum paths beginning from each. Following the bold arrows, the minimum paths are: 1-5-9-12-17, 4-7-9-12-17, and 4-7-11-14-16.



3. (a) The image X shown below contains four separate 8-connected blobs. Find the smallest structuring element B so that the image X(+)B (ie. X dilated by B) consists of just a single 8-connected blob. Smallest means least number of pixels in B. Express B as a set of pixels in ZxZ.

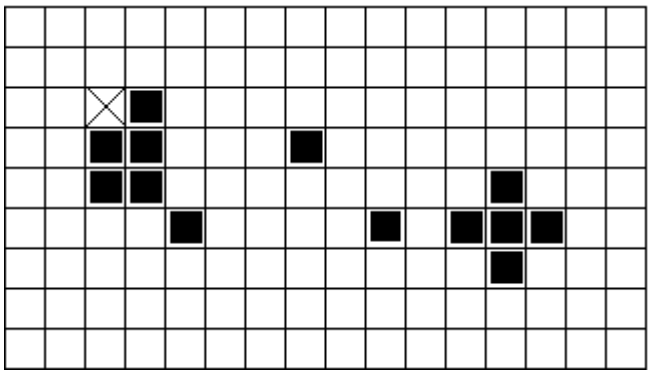


Image X for problem 3(a)

(b) Let B be the structuring element shown below. Give an example of any image X which, when opened using B, is unchanged, ie the opening of X using this B is just X itself. Then generalizing, state a set of necessary and sufficient conditions on an arbitrary image X for the opening of X using the B below to be just X itself.

Structuring element B for problem 3(b)

(a) B={(0,0) (1,-1) (3,-1)} is the smallest structuring element that dilates the given X into a single 8-connected blob. There are other solutions with 3 pixels also, which differ from the B given by an arbitrary shift of the origin. For instance if we add (-3,1) to each pixel in B, shifting the origin by (3,-1), we get B'={(-3,1) (-2,0) (0,0)}. Any such solution is equally good.

(b) Let X=B. Then X eroded by B will be a single pixel, and that single pixel, when dilated, will recover X. Generalizing, suppose X is the union of shifted copies of B. Then when eroded, each copy will reduce to a single pixel at the lower left hand corner of each copy, and when dilated, each copy of B will return to itself, and X will be unchanged. Now assume X is not the union of shifted copies of B. Then there is some pixel in X which is not part of a 2x2 square of black pixels. This pixel will be eliminated when eroded, and not return when dilated. So the opened version of X will be smaller than X itself. Thus the condition that X be the union of shifted copies of B is necessary and sufficient for X to be unchanged by the opening operation.