## Midterm Exam

Instructions: Answer all questions in the bluebook provided. closed book, notes. No electronic devices allowed.

1. Let $M$ be a $3 \times 3$ mask $M$ whose values are all -1 except the center pixel whose value is +8 . The origin of the mask is its center pixel.
(a) Find the $6 \times 6$ output when $M$ is applied to the $6 \times 6$ image shown below. Note: some of the values in the output will be negative.

| 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

(b) Suppose we apply M to a binary image, then we threshold the output so that only pixels +8 or higher are marked as foreground pixels. What image features in the original binary image does this procedure detect?
(c) Repeat (b) in the case where we change the threshold from +8 to +3 . What image features in the original binary image are detected?
2. Let X be an $8 \times 8$ binary image which contains exactly 23 foreground pixels.
(a) How many nodes are there in the T-Pyramid for X ?
(b) What is the minimum number of inner boundary pixels in any $8 \times 8$ image which contains exactly 23 foreground pixels? Use 8-connectivity to define adjacency between foreground and background pixels. Justify your answer.
(c) What is the maximum number of nodes in the quadtree for any $8 \times 8$ image with exactly 23 foreground pixels? Show how the 23 foreground pixels can be distributed so that the quadtree has this maximum number of nodes.
3. Define the limit of an image X as follows: given an image X , pick an operator $\Omega$, where $\Omega$ is one of the four operators dilation, erosion, opening, or closing. Next pick a structuring element B . Then Y is the limit of X using $\Omega$ and B if Y is the result of applying $\Omega$ using stucturing element B to the image X infinitely often:

$$
\mathrm{Y}=\mathrm{X} \Omega \mathrm{~B} \Omega \mathrm{~B} \Omega \mathrm{~B} \Omega \mathrm{~B} . . .
$$

An image X is shown in the upper left figure below. For each of the remaining 5 figures, specify an operator $\Omega_{\mathrm{i}}$ and structuring element $\mathrm{B}_{\mathrm{i}}$ for which $\mathrm{Y}_{\mathrm{i}}$ is the limit of X using $\Omega_{\mathrm{i}}$ and $\mathrm{B}_{\mathrm{i}}$. If you don't think there is $\Omega_{\mathrm{i}}$ and $\mathrm{B}_{\mathrm{i}}$ for which $\mathrm{Y}_{\mathrm{i}}$ is the limit of X using any $\Omega_{\mathrm{i}}$ or $\mathrm{B}_{\mathrm{i}}$, explain why not.
(a) Specify $\Omega_{1}$ and $\mathrm{B}_{1}$ so that $\mathrm{Y}_{1}$ is the limit of X using $\Omega_{1}$ and $\mathrm{B}_{1}$.
(b) Specify $\Omega_{2}$ and $\mathrm{B}_{2}$ so that $\mathrm{Y}_{2}$ is the limit of X using $\Omega_{2}$ and $\mathrm{B}_{2}$.
(c) Specify $\Omega_{3}$ and $\mathrm{B}_{3}$ so that $\mathrm{Y}_{3}$ is the limit of X using $\Omega_{3}$ and $\mathrm{B}_{3}$.
(d) Specify $\Omega_{4}$ and $\mathrm{B}_{4}$ so that $\mathrm{Y}_{4}$ is the limit of X using $\Omega_{4}$ and $\mathrm{B}_{4}$.
(e) Specify $\Omega_{5}$ and $\mathrm{B}_{5}$ so that $\mathrm{Y}_{5}$ is the limit of X using $\Omega_{5}$ and $\mathrm{B}_{5}$.


