

Equivalence Rules

Commutative: $A \vee B \Leftrightarrow B \vee A$ $A \wedge B \Leftrightarrow B \wedge A$

Associative: $(A \vee B) \vee C \Leftrightarrow A \vee (B \vee C)$ $(A \wedge B) \wedge C \Leftrightarrow A \wedge (B \wedge C)$

Distributive: $A \vee (B \wedge C) \Leftrightarrow (A \vee B) \wedge (A \vee C)$ $A \wedge (B \vee C) \Leftrightarrow (A \wedge B) \vee (A \wedge C)$

Identity: $A \vee \text{False} \Leftrightarrow A$ $A \wedge \text{True} \Leftrightarrow A$

Universal Bound: $A \vee \text{True} \Leftrightarrow \text{True}$ $A \wedge \text{False} \Leftrightarrow \text{False}$

Negation: $A \vee \sim A \Leftrightarrow \text{True}$ $A \wedge \sim A \Leftrightarrow \text{False}$

DeMorgan's: $\sim (A \vee B) \Leftrightarrow \sim A \wedge \sim B$ $\sim (A \wedge B) \Leftrightarrow \sim A \vee \sim B$

Absorption: $A \wedge (A \vee B) \Leftrightarrow A$ $A \vee (A \wedge B) \Leftrightarrow A$

Idempotent: $A \vee A \Leftrightarrow A$ $A \wedge A \Leftrightarrow A$

Double Negation: $\sim(\sim A) \Leftrightarrow A$

Implication: $A \rightarrow B \Leftrightarrow \sim A \vee B$

Equivalence: $A \leftrightarrow B \Leftrightarrow (A \rightarrow B) \wedge (B \rightarrow A)$

Inference Rules

Modus ponens

$$\begin{array}{l} P \rightarrow Q \\ \underline{P} \\ \therefore Q \end{array}$$

Modus tollens

$$\begin{array}{l} P \rightarrow Q \\ \underline{\sim Q} \\ \therefore \sim P \end{array}$$

Conjunction

$$\begin{array}{l} P \\ \underline{Q} \\ \therefore P \wedge Q \end{array}$$

Addition

$$\begin{array}{l} \underline{P} \\ \therefore P \vee Q \end{array}$$

Simplification

$$\begin{array}{l} \underline{P \wedge Q} \\ \therefore P \end{array} \qquad \begin{array}{l} \underline{P \wedge Q} \\ \therefore Q \end{array}$$

Hypothetical Syllogism

$$\begin{array}{l} P \rightarrow Q \\ \underline{Q \rightarrow R} \\ \therefore P \rightarrow R \end{array}$$

Disjunctive Syllogism

$$\begin{array}{l} P \vee Q \\ \underline{\sim P} \\ \therefore Q \end{array}$$

Contrapositive

$$\begin{array}{l} \underline{P \rightarrow Q} \\ \therefore \sim Q \rightarrow \sim P \end{array} \qquad \begin{array}{l} \underline{\sim Q \rightarrow \sim P} \\ \therefore P \rightarrow Q \end{array}$$

Exportation

$$\begin{array}{l} \underline{(P \wedge Q) \rightarrow R} \\ \therefore P \rightarrow (Q \rightarrow R) \end{array}$$

Inconsistency

$$\begin{array}{l} P \\ \underline{\sim P} \\ \therefore Q \end{array}$$