CSE 331: Introduction to Algorithm Analysis and Design Fall 2010

HOMEWORK 0
Do not turn in

This homework is to refresh your memory on stuff you should have seen in CSE 191 and/or CSE 250. This homework is intended for you to start thinking about proofs again. **Do not turn this one in, it will NOT be graded.**

No collaboration is allowed on the first problem but collaboration is allowed for the other two problems. For more details, see the Homework Policy document.

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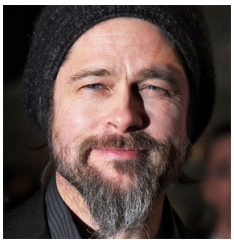
1. (**What is a proof?**) (40 points) Consider the following "proof":

   - Brad Pitt has a beard:[1]

   - Every goat has a beard:[2]

   - Hence, Brad Pitt is a goat.

   State precisely where the "proof" above fails logically.

   Follow-up question: Can you prove that Brad Pitt is not a goat?

2. (**Proof by Contradiction**) (45 points) Assume that the following are true:

   - Every `blockbuster movie` has a `hero`.
   - `Jake Sully` and `Neytiri` are dating.
   - The highest grossing `movie` ever is a `blockbuster`.

[1] Image Source: `http://www.waleg.com/celebrities/archives/019283.html`

[2] Image Source: Myers, P., R. Espinosa, C. S. Parr, T. Jones, G. S. Hammond, and T. A. Dewey. 2006. The Animal Diversity Web (online). Accessed August 28, 2010 at `http://animaldiversity.org`.

- A `hero` in a `movie` never `dies`.
- The `movie Avatar` has made the most money ever.
- The `heroine` always dates the `hero`.
- `Neytiri` is the `heroine` of `Avatar`.

Prove by contradiction that `Jake Sully` is `alive` at the end of the `movie Avatar`. Please clearly state any assumptions that you needed to make in the proof.

3. (**Induction**) (15 points) Let $n \geq 1$ be an integer. Given n men and n women, a *perfect matching* is a way to assign every man to a unique woman and vice-versa. For example for $n = 2$, where `Brad Pitt` and `Billy Bob Thornton` are the two men and `Jennifer Aniston` and `Angelina Jolie` are the two women, one possible perfect matching consist of the two pairs (`Brad Pitt`, `Jennifer Aniston`) and (`Billy Bob Thornton`, `Angelina Jolie`).

   Prove that the total number of perfect matchings when you have n men and n women is $n!$. (Recall $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$.)

   (*Hint:* Use induction. Also note that the number of perfect matching just depends on n and is *independent* of the identities of the men and women.)