

HOMEWORK 1

Due Friday, September 17, 2010 by 1:15pm in class

IMPORTANT: Please submit each problem separately, i.e. each problem should begin on a new page and only the pages for one problem should be stapled together. Failure to do so might result in some problem(s) not being graded.

For general homework policies and our suggestions, please see the homework policy document.

For this homework, it might help if you read the subsection “Extensions” in Section 1.1 of the textbook.

No collaboration is allowed on the first problem.

The third problem is due in **two weeks**, i.e. in class on Sep 24.

1. (**You must work on this problem on your own: NO collaboration is allowed**) (20 + 20 = 40 points) Exercise 1 and 2 in Chapter 1.
2. (45 points) Exercise 3 in Chapter 1.
3. (**Turn this problem in on Sep 24**) (15 points) For every $n \geq 1$, show that there is a stable matching instance on n men and n women such that the while loop in the Gale-Shapley algorithm runs $\Omega(n^2)$ times. Prove why your example works.

To get **full credit**, you should be able to present an instance for *every* $n \geq 1$. However, you have the freedom to decide how the Gale-Shapley algorithm chooses a free woman at any iteration of the while loop in case there is more than one choice. Make good use of this freedom: how you make the choice among the free women will be crucial in solving the problem.

(*Hint:* If it helps you in your thinking, one can construct a family of instances (and a run for the Gale-Shapley algorithm for each instance), where the while loop runs for at least $n(n+1)/2$ times.)