HOMEWORK 6
Due Friday, October 29, 2010 by 1:15pm in class

IMPORTANT: Please submit each problem separately, i.e. each problem should begin on a new page and only the pages for one problem should be stapled together. Failure to do so might result in some problem(s) not being graded.
For general homework policies and our suggestions, please see the policy document.
No collaboration is allowed on problem 1.

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1. (40 points) **Your must work on this problem on your own: NO collaboration is allowed**) You are given n jobs such that for job i we are given a pair (s_i, e_i), where s_i is the start time for job i and e_i is the end time. Two jobs i and j conflict if $\{s_i, \ldots, e_i - 1\} \cap \{s_j, \ldots, e_j\} \neq \emptyset$. A schedule is a subset of the jobs such that no two jobs in the schedule conflict. Present an $O(n \log n)$ time algorithm, which given the set of jobs and a number $0 \leq k \leq n$ as input outputs "yes" if there exists a schedule of the jobs with at least k elements in it else it outputs "no."

2. (45 points) Exercise 3 in Chapter 4.

   Note: In this problem you are *given* the algorithm. The problem is asking you to analyze the algorithm using an analysis similar to the one we saw in class for Interval Scheduling. It might help to mathematically formulate the problem first. So some fraction of the grade will be allotted to the mathematical formulation of the problem. Also, in many real life problems, not all parameters are equally important, so keep that in mind when tackling this question.

3. (15 points) Ms. LiberalElite likes driving her Prius, which on one full tank of gas runs for 350 miles.[1] She is planning to drive from Buffalo to Seattle and has fixed up her driving route so that she can get to Seattle as fast as possible. She also has a map with the locations of all the n gas stations along the way. You can assume that the route is a straight line and the gas stations are points on the line. Ms. LiberalElite missed the class on greedy algorithms, so in this problem you will design an efficient algorithm for her, which she can use to figure out which gas stations she should stop at so that she stops at the minimum number of gas stations. Of course your algorithm should give a feasible set of gas stations, i.e. it should never be the case that is she is stranded between two gas stations without any gas. (You may assume that Ms. LiberalElite fills up the tank of her Prius whenever she stops at a gas station and that no two gas stations are more than 349 miles apart. Also you can assume that she starts off from Buffalo with a full tank.)

   Prove the correctness of your algorithm and analyze its running time. (For the latter, you will get more credit the smaller the running time of your algorithm.)

   Hint: Think of a greedy way to decide on which gas stations to stop. It might help to forget about the scheduling algorithms we have seen in class and just think of a greedy algorithm

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[1]Lest you think I have anything against liberals or Prius, let me assure you that my brother-in-law has "termed" me and my wife as being liberal elites. And yes, we do have a Prius.

from "scratch." Then try to analyze the algorithm using similar arguments to one we have seen in class.