CSE 331: Introduction to Algorithm Analysis and Design Fall 2010

HOMEWORK 7

Due Friday, November 5, 2010 by 1:15pm in class

IMPORTANT: Please submit each problem separately, i.e. each problem should begin on a new page and only the pages for one problem should be stapled together. Failure to do so might result in some problem(s) not being graded.
For general homework policies and our suggestions, please see the policy document.

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1. (40 points) Part (b) of Exercise 2 in Chapter 4.

2. (45 points) Mr. Greedy Barber has to schedule n clients for haircuts for the next day. These n clients are regular customers, so for each customer i, Mr. Barber know the exact time h_i it would take him to cut i's hair. Mr. Barber is also gunning for a record, so he has decided not to waste any time moving from one haircut to the next. Help Mr. Barber design an $O(n \log n)$ time algorithm that schedules all the n clients, which provably minimizes the *sum* of the *service time* for every client, where the service time for client i is h_i plus the time i had to wait before i's turn came by. You should assume that all the clients came into the shop at the same time. As usual, you must prove the optimality of your algorithm.

   Here is a simple example: say $n = 3$ and

   $$h_1 = 5, \quad h_2 = 10, \quad h_3 = 4.$$

   Now consider the schedule

   $$1, 2, 3$$

   i.e. 1 get the haircut first and then leaves, 2 gets his haircut after 1 and 3 gets his hair cut after 2. Note that the service time for 1 is 5. The service time for client 2 is his wait time (which is 5) and h_2, which is 15. The service time for client 3 is his wait time (which is $5 + 10 = 15$) plus h_3, which is 19. Thus, the sum of the service times for this schedule is $5 + 15 + 19 = 39$. This schedule, however, is not optimal.

3. (15 points) Exercise 6 in Chapter 4.

   Note: As mentioned in the previous homework, in many real life problems, not all parameters are equally important. Also sometimes it might make sense to "combine" two parameters into one. Keep these in mind when tackling this problem.

   Hint: In the solution that I have in mind, the analysis of the algorithm's correctness follows the exchange argument.