CSE 331: Introduction to Algorithm Analysis and Design Fall 2011

HOMEWORK 1

Due Friday, September 16, 2009 by 1:15pm in class

IMPORTANT: Please submit each problem separately, i.e. each problem should begin on a new page and only the pages for one problem should be stapled together. Failure to do so might result in some problem(s) not being graded.

For general homework policies and our suggestions, please see the homework policy document. **You can and will lose points if you do not follow the policies.**

For this homework, it might help if you read the the subsection "Extensions" in Section 1.1 of the textbook.

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1. ($20 + 20 = 40$ points)

   (a) Decide whether the following statement is true or false:

   > In every Stable Marriage problem instance, there always exists a stable matching where for every matched pair (m, w), it is true that both m and w do not have the other as their least preferred partner.

   If you state true then you will have to formally argue why the statement is correct. If you state false, then you have give a counter-example.

   (b) Exercise 2 in Chapter 1.

2. (45 points) Exercise 4 in Chapter 1.

3. (15 points) This problem is inspired by a question raised by Max in class. Max mentioned that in real life there are divorces and I mentioned that the stable marriage problem does not handle divorces. This is because we assume everyone is interested in everyone else of the opposite sex and we assume that the preferences *do not change*.

   In this problem, we will see the effect of changes in preferences in the outcome of the Gale-Shapley algorithm (for this problem you can assume the version of the Gale-Shapley algorithm that we did in class where the women do all the proposing).

   Given an instance of the stable marriage problem (i.e. set of men M and the set of women W along with their preference lists: L_m and L_w for every $m \in M$ and $w \in W$ respectively), call a man $m \in M$ a *home-wrecker* if the following property holds. There exists an L'_m such that if m changes his preference list to L'_m (from L_m) then the Gale-Shapley algorithm matches everyone to someone else. In other words, let S_{orig} be the stable marriage output by the Gale-Shapley algorithm for the original input and S_{new} be the stable marriage output by the Gale-Shapley algorithm for the new instance of the problem where m's preference list is replaced by L'_m (but everyone else has the same preference list as before). Then $S_{\text{orig}} \cap S_{\text{new}} = \emptyset$.

   For every integer $n \geq 2$ prove the following: There exists an instance of the stable marriage problem with n men and n women such that there is a man who is a home-wrecker.

   (*Note*: To get full credit you must present an example for every $n \geq 2$, that is, you have to present a "family" of examples. Further, your proof argument should work for every value of $n \geq 2$.)