Def: Graph

$$G = (V, E)$$

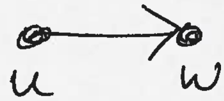
set of vertices/nodes (V) — set of edges (E)

$$E \subseteq V \times V$$

Types

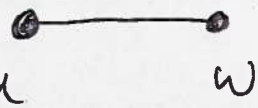
(i) Directed graph

$(u,w) \in E \not\Rightarrow (w,u) \in E$ (not necessarily imply)



(ii) Undirected graph

$$(u,w) \in E \Rightarrow (w,u) \in E$$



Ex:
(i) TV host example: undirected
(ii) Internet: undirected
(iii) Facebook: Undirected
(iv) Wikipedia: Directed.
(v) Airline routes: Undirected

$\equiv$

Every undirected graph is directed

Assumptions: Unless stated o/w

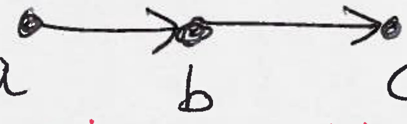
(i) $(u,u) \notin E$ (ii) a graph will be undirected

$G = (V, E)$

Def: Path sequence

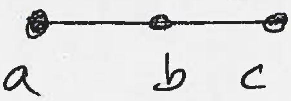
$u_1, \ldots, u_k \qquad u_i \in V$

$(u_i, u_{i+1}) \in E \qquad 1 \le i \le k-1$

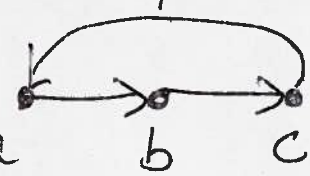


(path fr a to c but not c to a)

a is connected to c but c is not connected to a.



(both paths exist)



a, b, a, b, c ✗

Simple path: no vertices are repeated.

a, b, c is ✓

Assumption: A path is a simple path.

Def: u, w is connected iff $\exists$ a path from u to w. (undirected graph)

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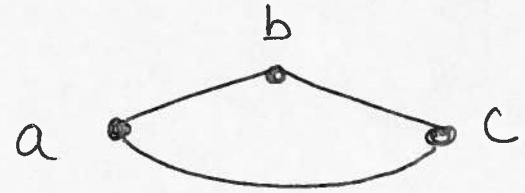
u, w strongly connected iff $\exists$ $u \rightsquigarrow w$ & $w \rightsquigarrow u$

Def: G is (strongly) connected iff every pair of vertices u & w are (strongly connected)

Def: Path length of $u_1, \dots, u_k$ is $k-1$.



distance between u & w is the shortest length of any $u \sim w$ path

Def: Cycle $(k \geq 3) \rightsquigarrow$ undirected (directed $k \geq 2$)

$u_1, \dots, u_k$ $\quad u_1, \dots, u_{k-1}$ are distinct

$u_k = u_1$

$(u_i, u_{i+1}) \in E$