THM. Any 2 out of 3 conditions below $\Rightarrow$ 3rd cond.

(G is undirected)

(1) G is ~~undirected~~ & connected

(2) G has no cycles

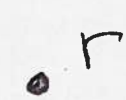
(3) G has n-1 edges

---

(1) + (2) $\Rightarrow$ (3)

Lemma: A tree T on n vertices has exactly n-1 edges

Pf. idea: Pick $T = (V, E)$ $r \in V$ as root



& root T at r

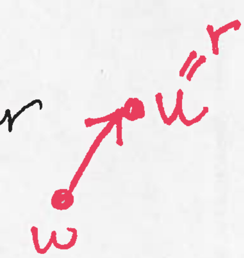
$\rightarrow$ For each edge (u, w)

orient the edge towards the vertex closer to r.

Claim 1: Every non-root vertex u, has exactly 1 outgoing edge

Claim 2: r has 0 outgoing edge. ✓

Claim 3: Every edge is outgoing edge for exactly one non-root vertex. ✓



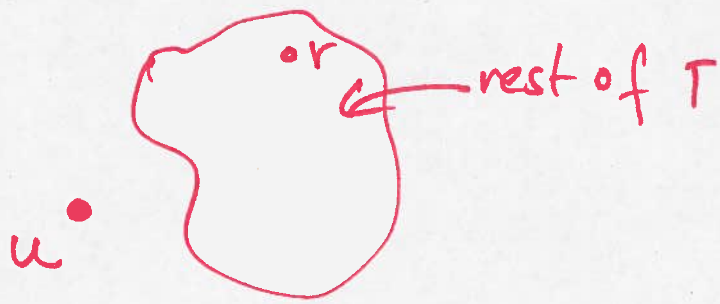
$\Rightarrow$ $|E| = \#$ non-root vertices $= n-1$

Pf of Claim 1: Consider $u \in V \setminus \{r\}$

Case 1: u has exactly 1 outgoing edge

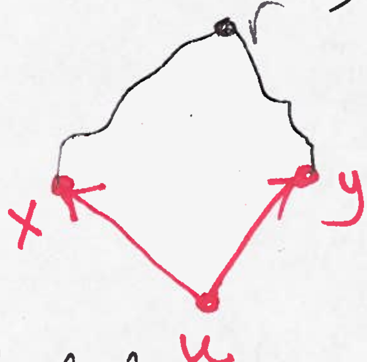
Case 2: u has exactly 0 outgoing edges

$\Rightarrow$



Contradiction, as this $\Rightarrow$ T is not connected.

Case 3: u has ≥ 2 outgoing edges.



(1) as T is connected $\Rightarrow \exists\ x \sim r$ & $y \sim r$

$\Rightarrow$ Contradiction $\Rightarrow$ T has a cycle. $\blacksquare$

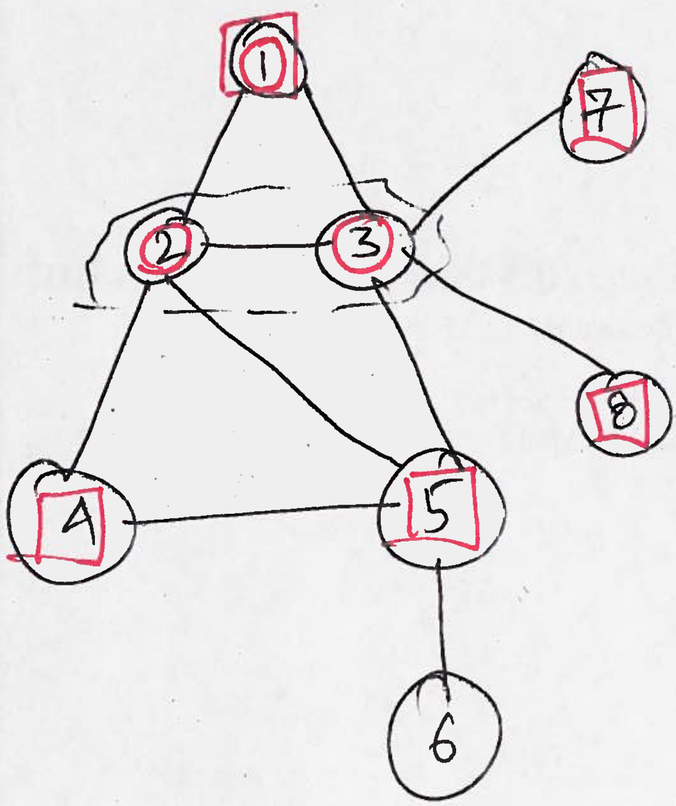
I/P: $G = (V, E)$, $s \in V$

O/P: All $w \in V$ s.t. s & w are connected.

$\Rightarrow$ can solve the s,t-connectivity problem.

$s = 1$.

Answer: $\{1, \ldots, 8\}$



$\rightarrow$ Output s first

$\rightarrow$ Output all u s.t. $(s, u) \in E$

$\rightarrow$ Output all friends of friend of s.

Assumption: given $u \in V$, easy to compute all w s.t. $(u, w) \in E$