Lemma: There $\exists$ an optimal schedule with 0 idle time & 0 # inversions.

Pf idea "Exchange argument"

① $L(\theta_{i+1}) \le L(\theta_i)$

② $\#inv(\theta_{i+1}) = \#inv(\theta_i) - 1$

0 idle time

$$\theta_0 \longrightarrow \theta_1 \longrightarrow \theta_2 \cdots \longrightarrow \theta_i \longrightarrow \theta_{i+1} \longrightarrow \cdots \theta^*$$

θ_0: optimal, 0 idle time

θ_i: optimal, 0 idle time

θ_{i+1}: optimal

max # inversions $\le \binom{n}{2}$ ← as only ≤ 1 one of (i,j) & (j,i) can be an inversion.

θ^*: optimal, 0 idle time, 0 # inversion

Existence of the black-box:

$$\theta \longrightarrow \boxed{} \longrightarrow \theta'$$

① idle time(θ') = idle time(θ) ~~=~~ = 0 true

② $L(\theta') \le L(\theta)$

③ $\#inv(\theta') = \#inv(\theta) - 1$ unless $\#inv(\theta) = 0$

→ θ has ≥ 1 inversion.
(i,j) is an inversion.

(a) $\exists$ inversion (i,j) s.t.

i	j

← θ

Swap i & j to get θ'

j	i

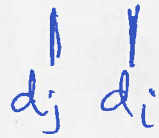
d_j d_i

(b) $\#inv(\theta') = \#inv(\theta) - 1$

(c) $L(\theta') \le L(\theta)$

Pf of (a) Let ~~us~~ say (i, j) is an inversi

Case 0: j is right after i'

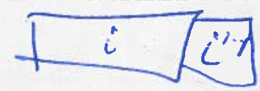


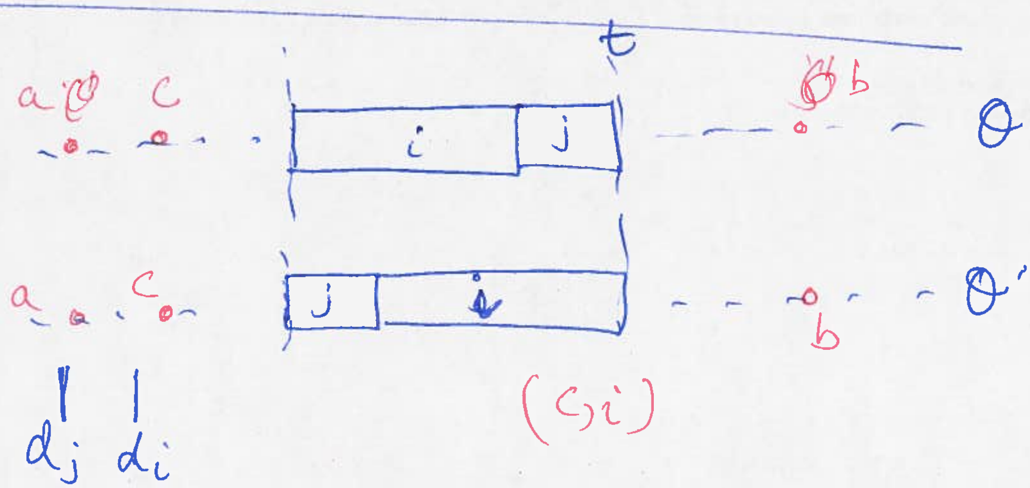
Case 1: $d_{i'} > d_j \Rightarrow$ done!

Case 2: $d_{i'} \le d_j < d_i \Rightarrow d_{i'} < d_i$

$\Rightarrow (i, i')$ is an inversion.

Repeat this $\le n$ times to get





(b) (i, j) is an inv in θ but not in θ'

Obs: $\forall (a, b) \neq (i, j)$

(a, b) is an inv in θ

$\Leftrightarrow (a, b)$ is an inv in θ'

part (c) lateness of any $k \notin \{i, j\}$ is the same

① $l_j^{\theta'} \le l_j^{\theta}$ ② $\boxed{l_i^{\theta'} \le l_i^{\theta}}$ X not true

①' $\max\{l_i^{\theta'}, l_j^{\theta'}\} \le \max\{l_i^{\theta}, l_j^{\theta}\} = l_j^{\theta}$

②' $l_i^{\theta'} \le l_j^{\theta}$

$$l_i^{\theta'} = t - d_i \le t - d_j = l_j^{\theta}$$

$d_i > d_j$