Lecture 12

CSE 331

Sep 22, 2014

Today's agenda

Finish Proving n vertex tree has n-1 edges

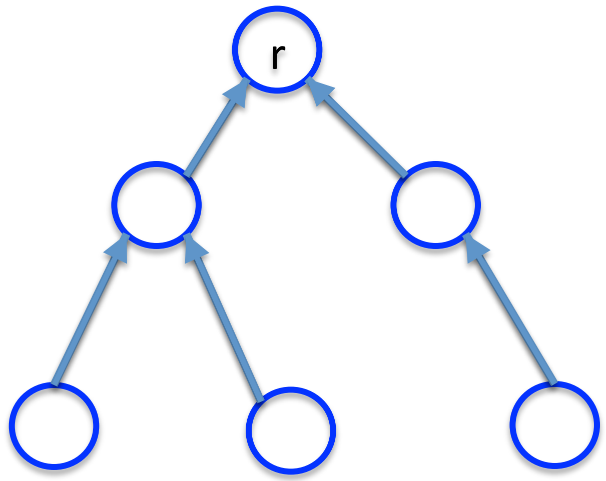
Algorithms for checking connectivity

n vertex tree has n-1 edges

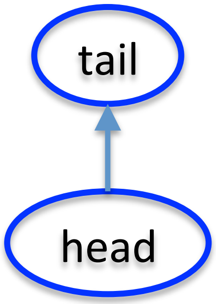
Proof idea (recap):

Root tree T at vertex r

Direct edges from child to parent



n vertex tree has $n-1$ edges

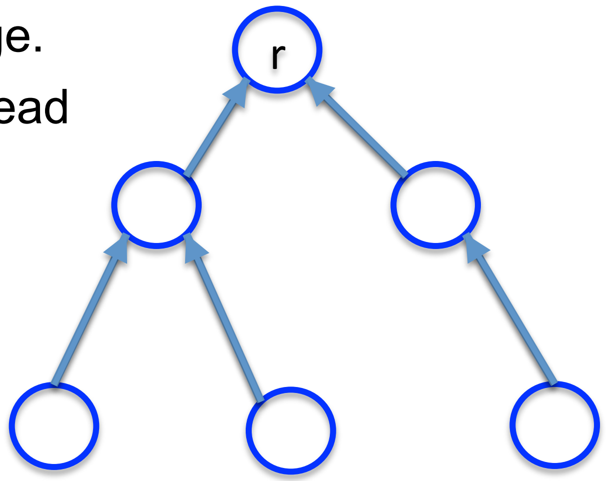


Proof idea (recap):

Claim 1: r is not the head of any directed edge.

Claim 2: Every directed edge has a unique head

Claim 3: Every non-root $u \in V$ is the head of some directed edge.



Claim 4: Every $u \in V$ is the head of ≤ 1 edges.

Claim 1-4 $\Rightarrow$ 1-to-1 correspondence between E and $V \setminus \{r\}$
$\Rightarrow |E| = |V \setminus \{r\}| = n - 1$

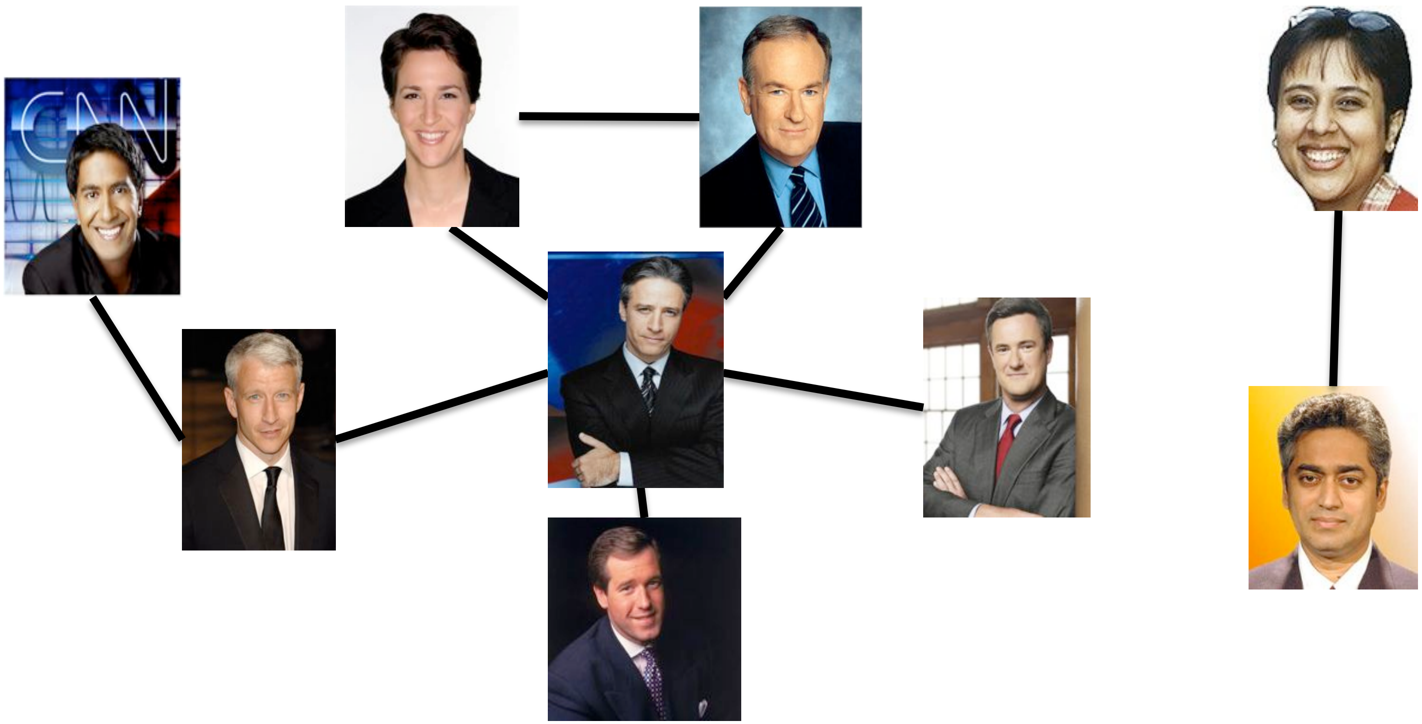
Questions?

Rest of Today's agenda

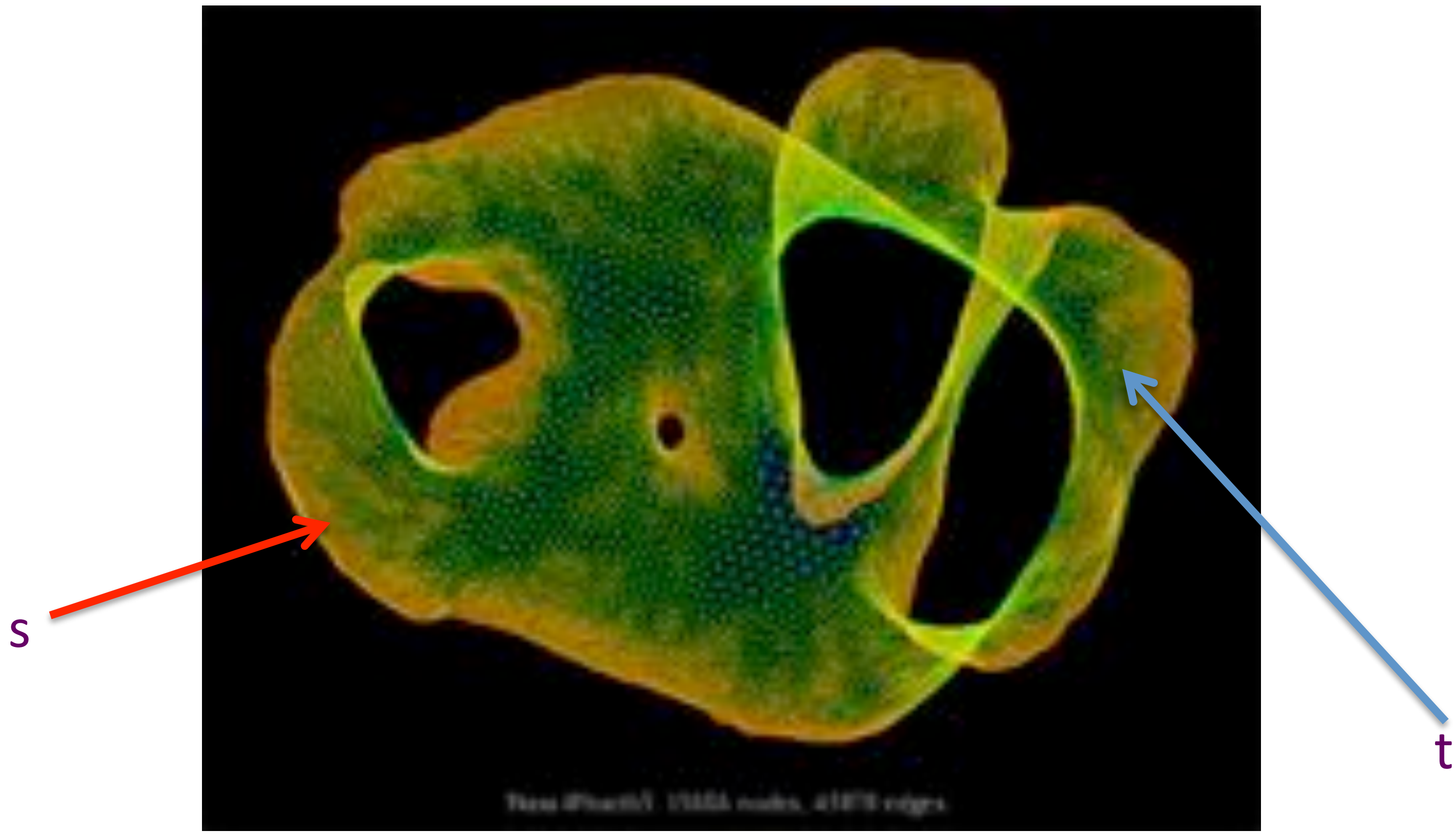
Finish Proving n vertex tree has n-1 edges

Algorithms for checking connectivity

Checking by inspection



What about large graphs?



Are s and t connected?

Brute-force algorithm?

List all possible vertex sequences between s and t

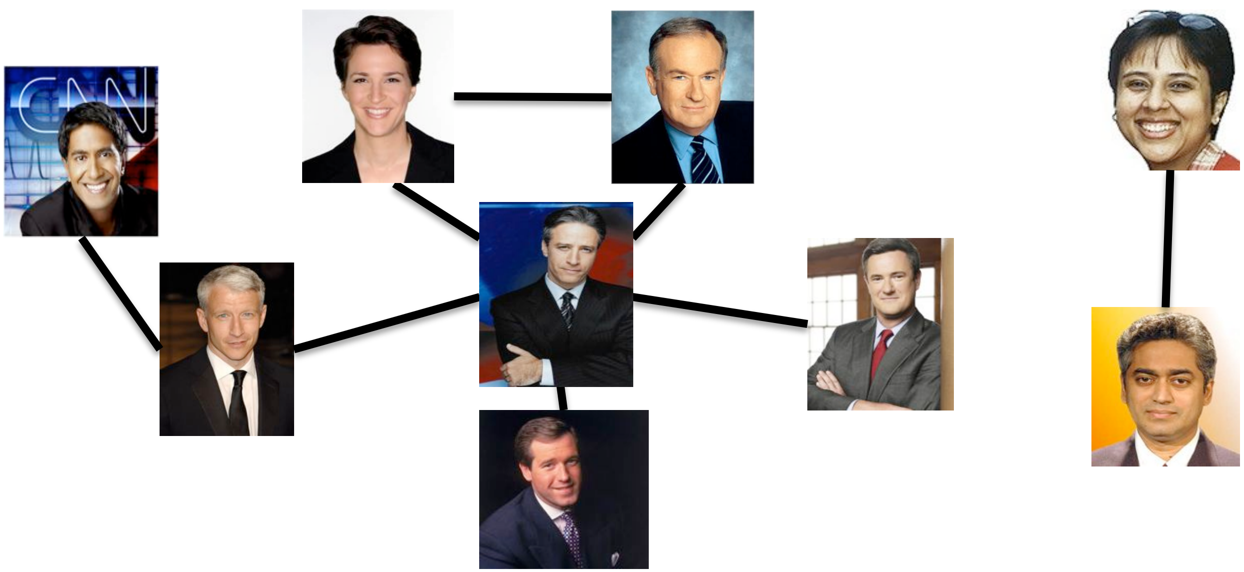
n^n such sequences

Check if any is a path between s and t

Algorithm motivation

Distance between u and v

Length of the shortest length path between u and v



Distance between RM and BO? 1

Questions?

Breadth First Search (BFS)

Is s connected to t?

Build layers of vertices connected to s

L_j : all nodes at distance j from s

$L_0 = \{s\}$

Assume $L_0,..,L_j$ have been constructed

L_{j+1} set of vertices not chosen yet but are connected to L_j

Stop when new layer is empty

Exercise for you

Prove that L_j has all nodes at distance j from s

BFS Tree

BFS naturally defines a tree rooted at s

L_j forms the jth "level" in the tree

u in L_{j+1} is child of v in L_j from which it was "discovered"