Lecture 27

CSE 331

Oct 31, 2014

HW 6 due today

Place Q1, Q2 and Q3 in separate piles

I will not accept HWs after 1:15pm

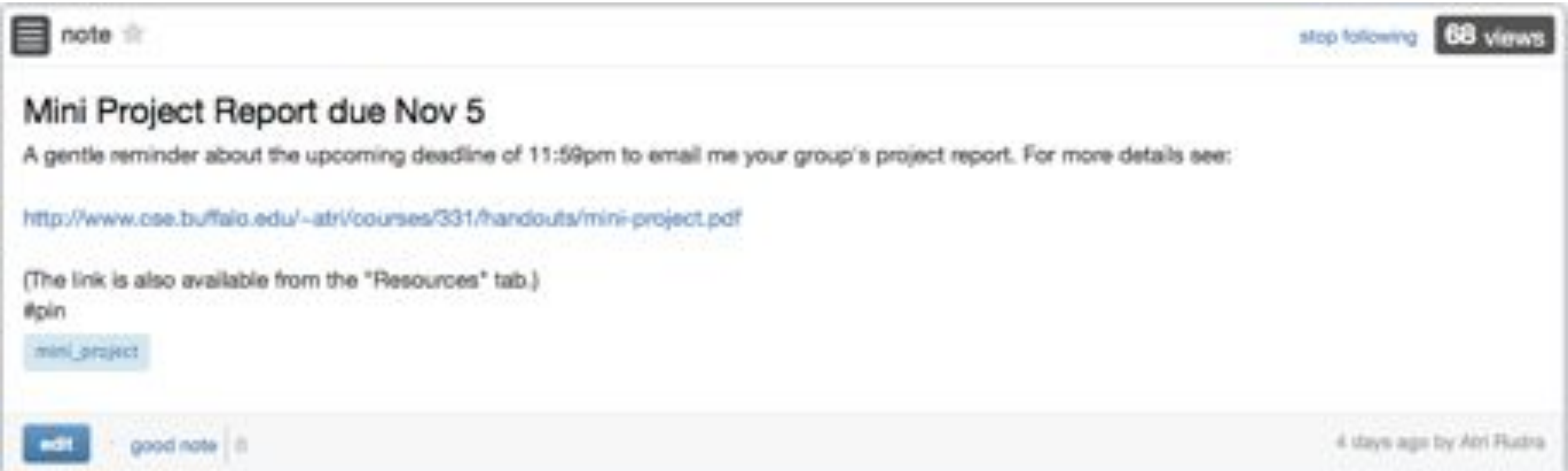
HW 7 on piazza

NO COLLABORATIONS ON Q1

HW 6 solutions

END of the lecture

Mini project report due WED


note

stop following

68 views

Mini Project Report due Nov 5

A gentle reminder about the upcoming deadline of 11:59pm to email me your group's project report. For more details see:

http://www.cse.buffalo.edu/~atri/courses/331/handouts/mini-project.pdf

(The link is also available from the "Resources" tab.)
#pin

mini_project

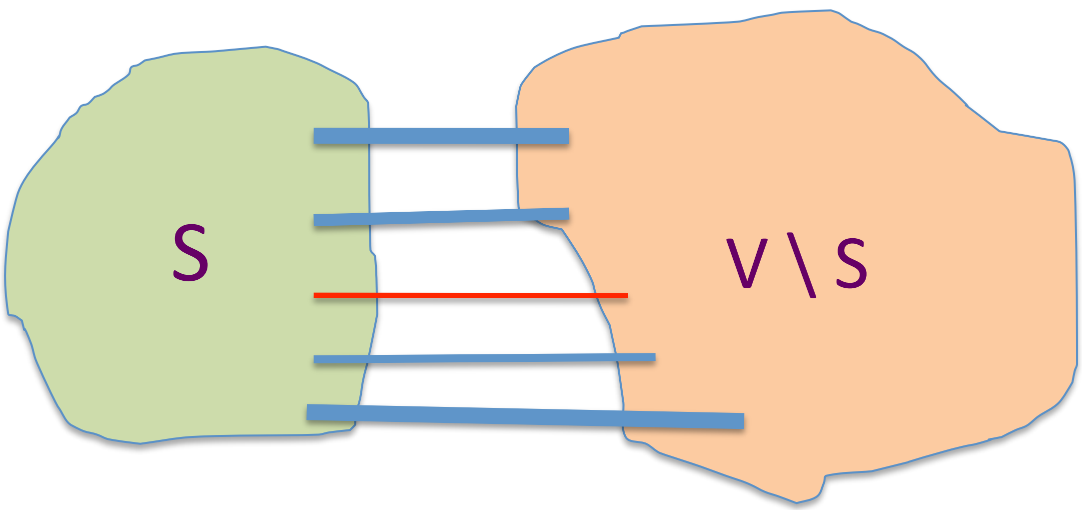
edit

good note

4 days ago by Atri Rudra

Cut Property Lemma for MSTs

Condition: S and V\S are non-empty

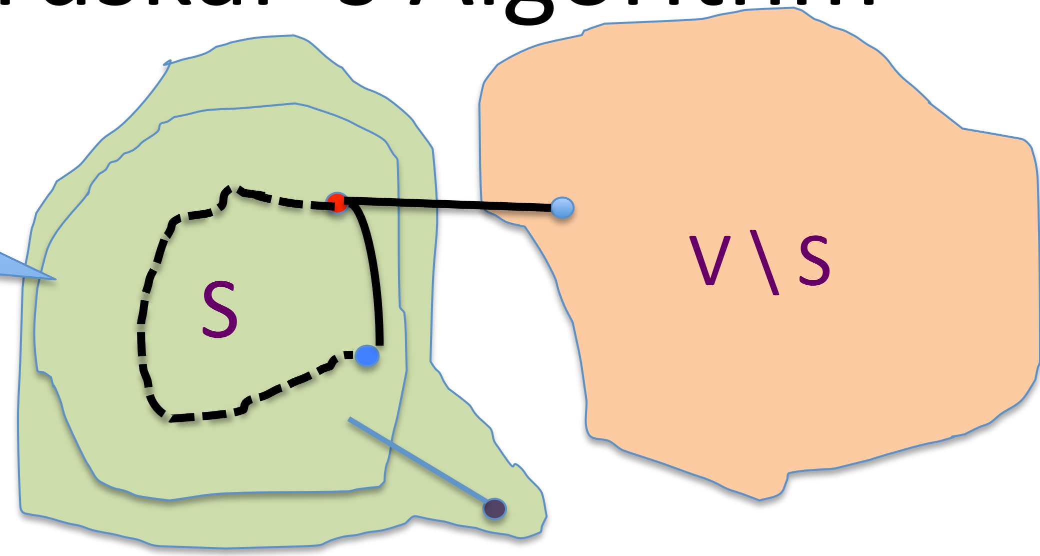


Cheapest crossing edge is in **all** MSTs

Assumption: All edge costs are distinct

Optimality of Kruskal's Algorithm

Nodes connected to red in (V,T)



Input: G=(V,E), $c_e > 0$ for every e in E

T = Ø

Sort edges in increasing order of their cost

Consider edges in sorted order

If an edge can be added to T without adding a cycle then add it to T

S is non-empty

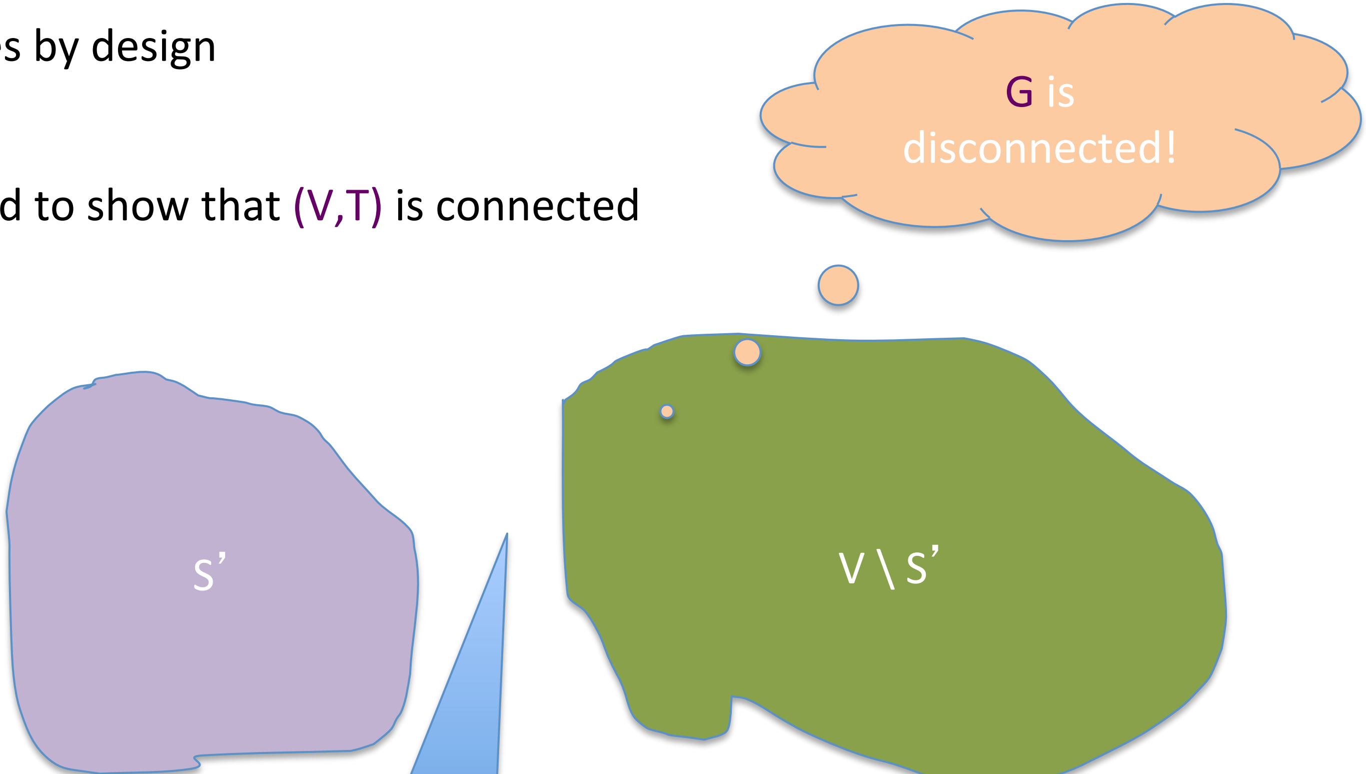
V\S is non-empty

First crossing edge considered

Is (V,T) a spanning tree?

No cycles by design

Just need to show that (V,T) is connected



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Removing distinct cost assumption

Change all edge weights by very small amounts

Make sure that all edge weights are distinct

MST for "perturbed" weights is the same as for original

Changes have to be small enough so that this holds

EXERCISE: Figure out how to change costs

Running time for Prim's algorithm

Similar to Dijkstra's algorithm

O(m log n)

Input: G=(V,E), $c_e > 0$ for every e in E

S = {s}, T = Ø

While S is not the same as V

Among edges e= (u,w) with u in S and w not in S, pick one with minimum cost

Add w to S, e to T

Running time for Kruskal's Algorithm

Can be implemented in O(m log n) time (Union-find DS)

Input: $G=(V,E)$, $c_e > 0$ for every e in E

O(m^2) time overall

$T = \emptyset$

Sort edges in increasing order of their cost

Consider edges in sorted order

If an edge can be added to T without adding a cycle then add it to T

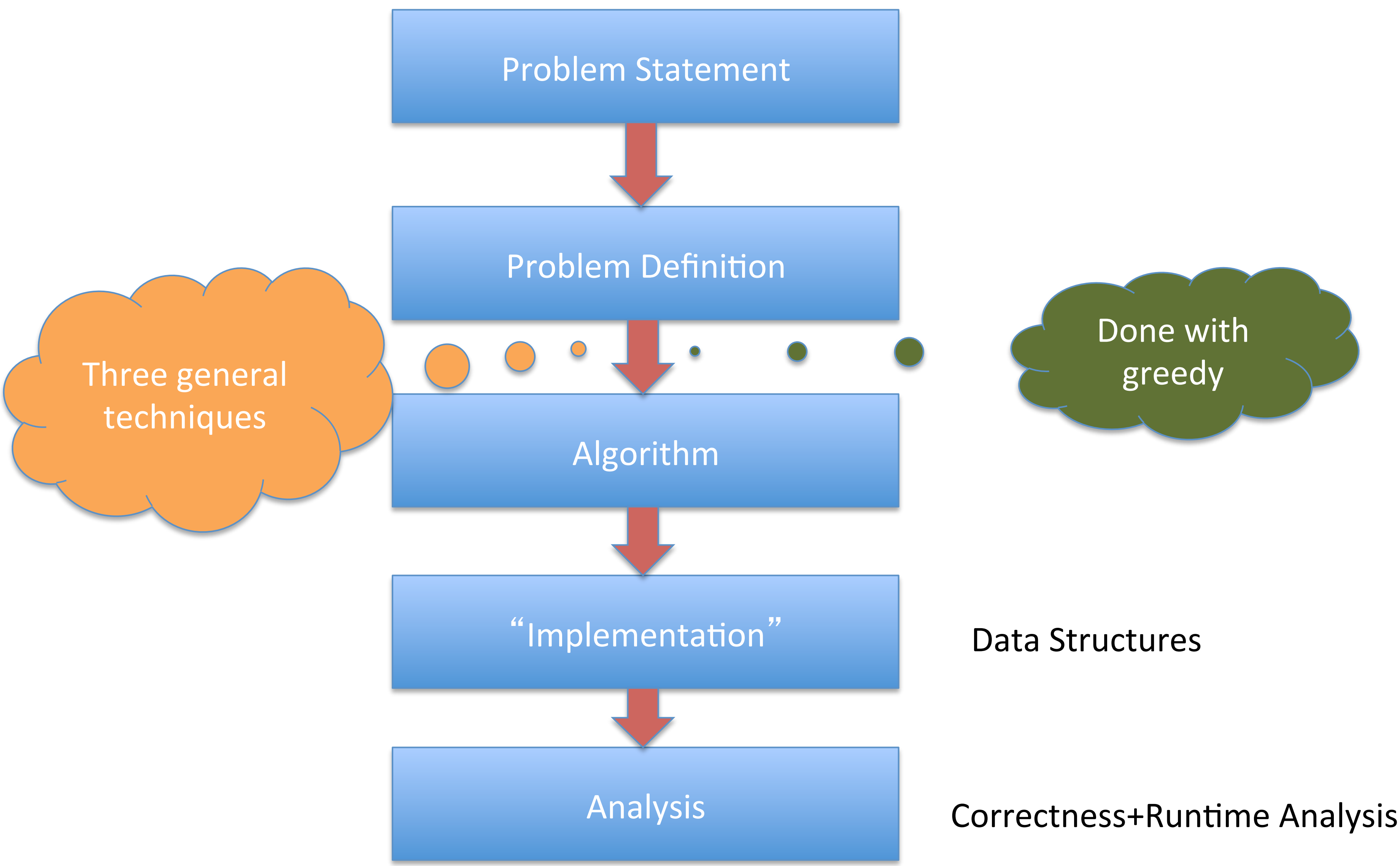
Can be verified in O(m+n) time

Joseph B. Kruskal

Reading Assignment

Sec 4.5, 4.6 of [KT]

High Level view of the course



Trivia

Divide and Conquer

Divide up the problem into at least two sub-problems

Recursively solve the sub-problems

“Patch up” the solutions to the sub-problems for the final solution

Sorting

Given n numbers order them from smallest to largest

Works for any set of elements on which there is a total order

Insertion Sort

Input: $a_1, a_2, \ldots, a_n$

Output: $b_1, b_2, \ldots, b_n$

$O(n^2)$ overall

Make sure that all the processed numbers are sorted

$b_1 = a_1$

for $i = 2 \ldots n$

  Find $1 \leq j \leq i$ s.t. a_i lies between b_{j-1} and b_j

  Move b_j to b_{i-1} one cell "down"

  $b_j = a_i$

O(log n)

O(n)

a	b
4	[illegible]
3	[illegible]
2	[illegible]
1	4

Other $O(n^2)$ sorting algorithms

Selection Sort: In every round pick the min among remaining numbers

Bubble sort: The smallest number "bubbles" up

Divide and Conquer

Divide up the problem into at least two sub-problems

Recursively solve the sub-problems

"Patch up" the solutions to the sub-problems for the final solution

Mergesort Algorithm

Divide up the numbers in the middle

Unless n=2

Sort each half recursively

Merge the two sorted halves into one sorted output

How fast can sorted arrays be merged?

Group talk time

Mergesort algorithm

Input: $a_1, a_2, \ldots, a_n$

Output: Numbers in sorted order

MergeSort(a, n)

If $n = 1$ **return** the order a_1

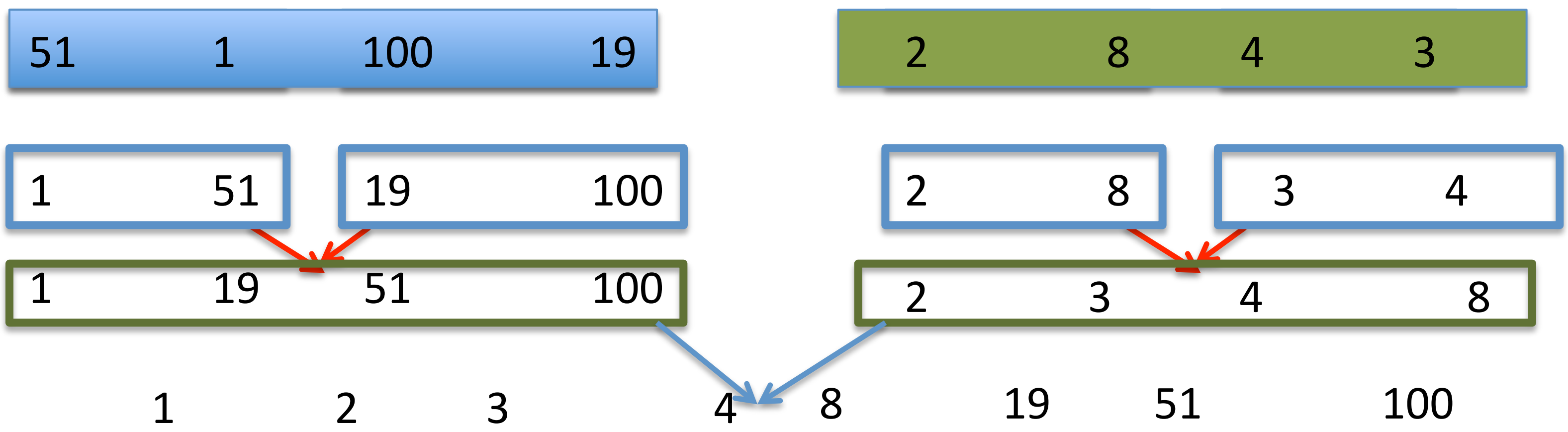
If $n = 2$ **return** the order $\min(a_1,a_2); \max(a_1,a_2)$

$a_L = a_1,\ldots, a_{n/2}$

$a_R = a_{n/2+1},\ldots, a_n$

return MERGE (MergeSort(a_L, $n/2$), MergeSort(a_R, $n/2$))

An example run



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If $n = 1$ **return** the order a_1

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return MERGE (MergeSort(a_L, $n/2$), MergeSort(a_R, $n/2$))

Correctness

Input: $a_1, a_2, \ldots, a_n$

Output: Numbers in sorted order

MergeSort(a, n)

If $n = 1$ **return** the order a_1

If $n = 2$ **return** the order $\min(a_1,a_2); \max(a_1,a_2)$

$a_L = a_1, \ldots, a_{n/2}$

$a_R = a_{n/2+1}, \ldots, a_n$

return MERGE (MergeSort(a_L, n/2), MergeSort(a_R, n/2))

By induction on n

Inductive step follows from correctness of MERGE