Lecture 32

CSE 331

Nov 12, 2014

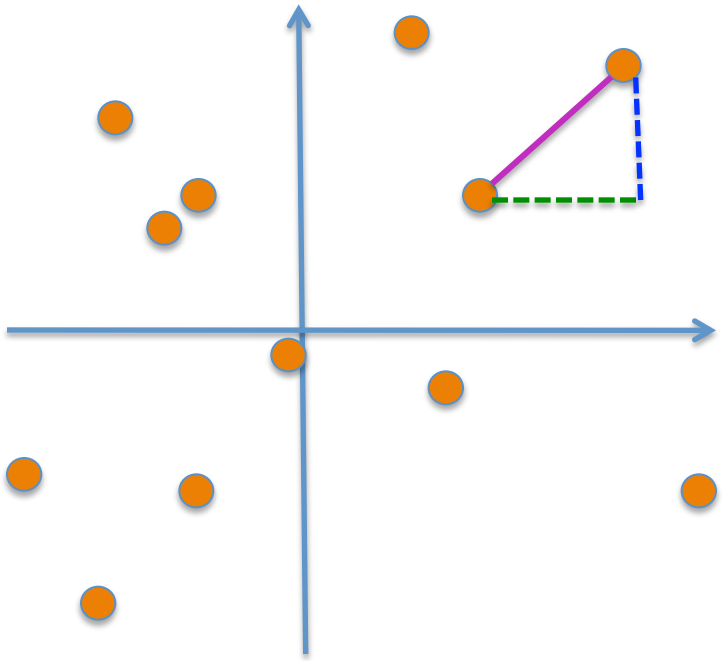
Online OH tonight @8:45pm

Closest pairs of points

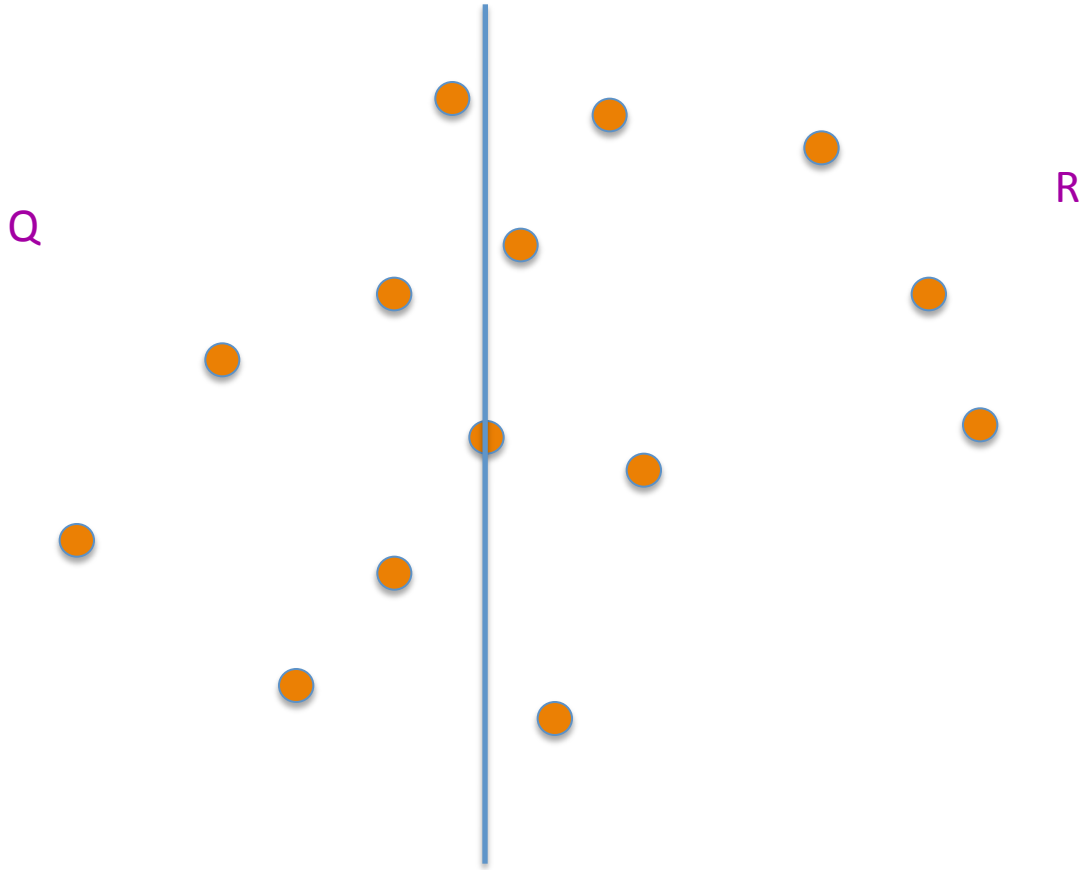
Input: n 2-D points $P = \{p_1,\ldots,p_n\}$; $p_i=(x_i,y_i)$

$d(p_i,p_j) = ((x_i-x_j)^2+(y_i-y_j)^2)^{1/2}$

Output: Points p and q that are closest

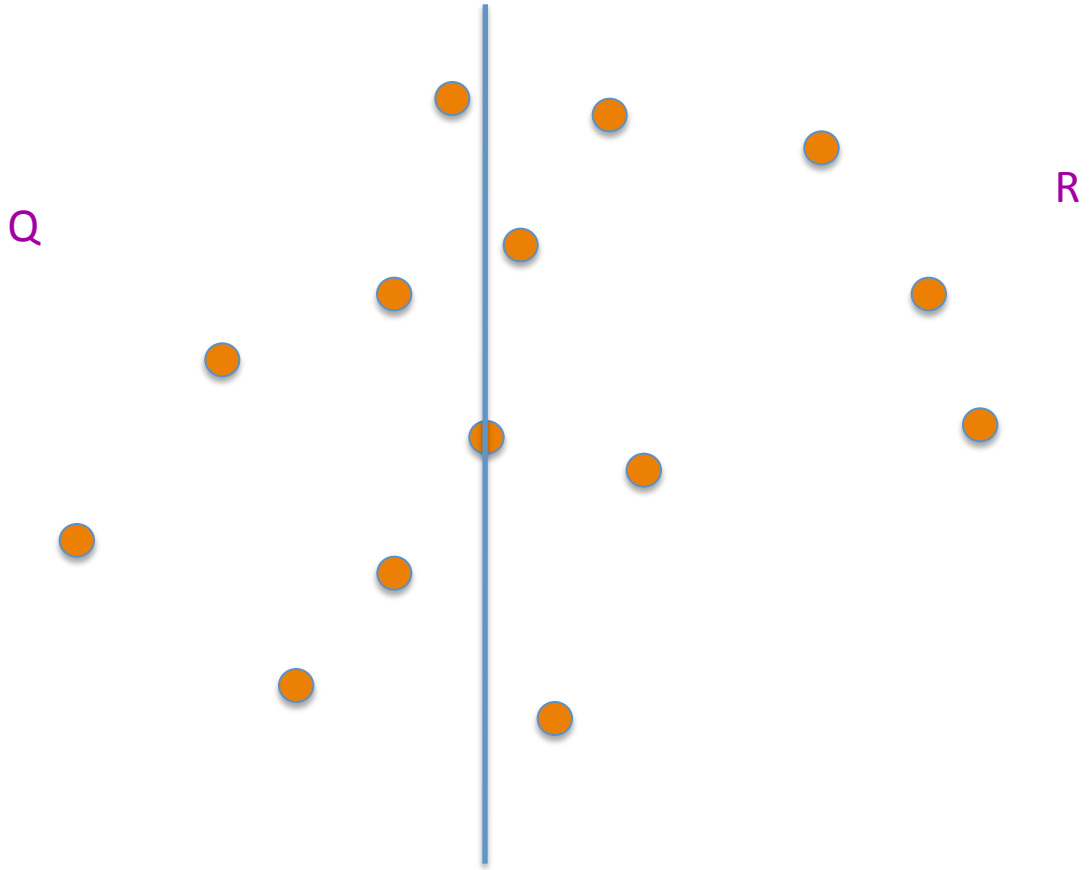


Dividing up P



First n/2 points according to the x-coord

Recursively find closest pairs



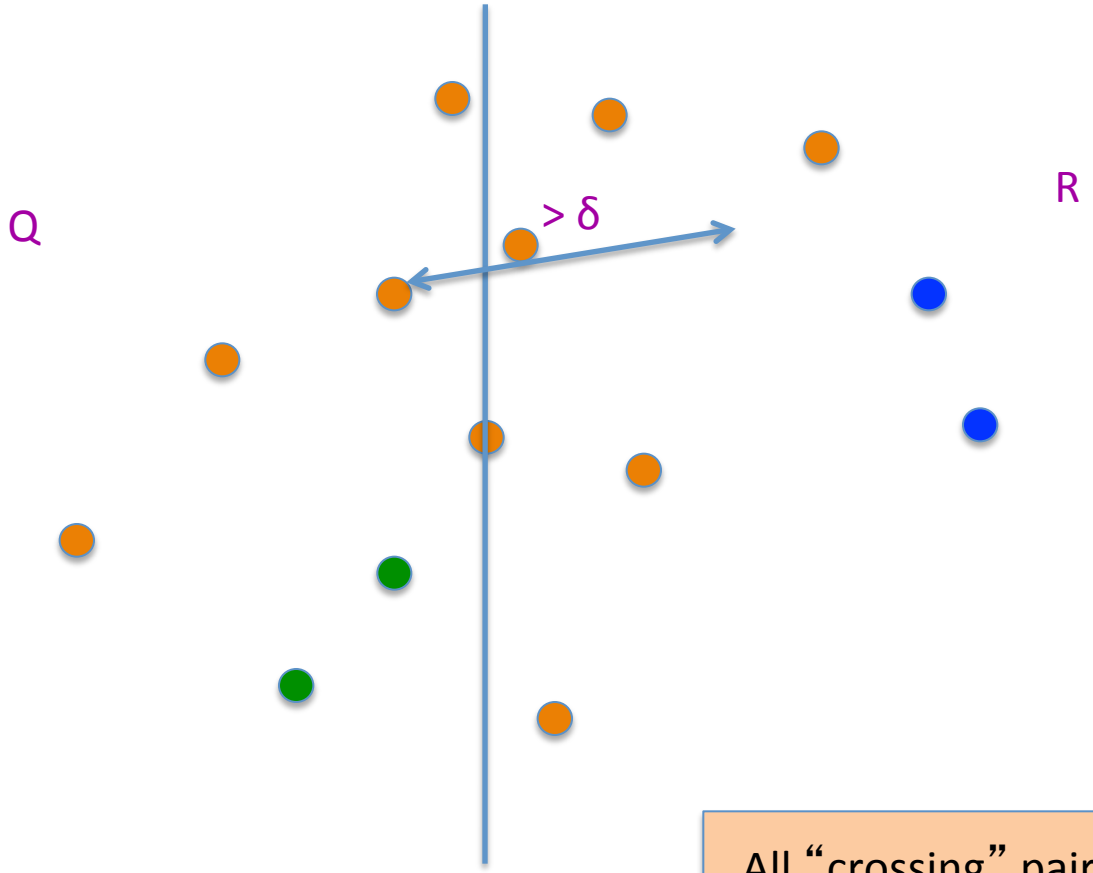
δ = min (**blue**, **green**)

An aside: maintain sorted lists

P_x and P_y are P sorted by x-coord and y-coord

Q_x, Q_y, R_x, R_y can be computed from P_x and P_y in $O(n)$ time

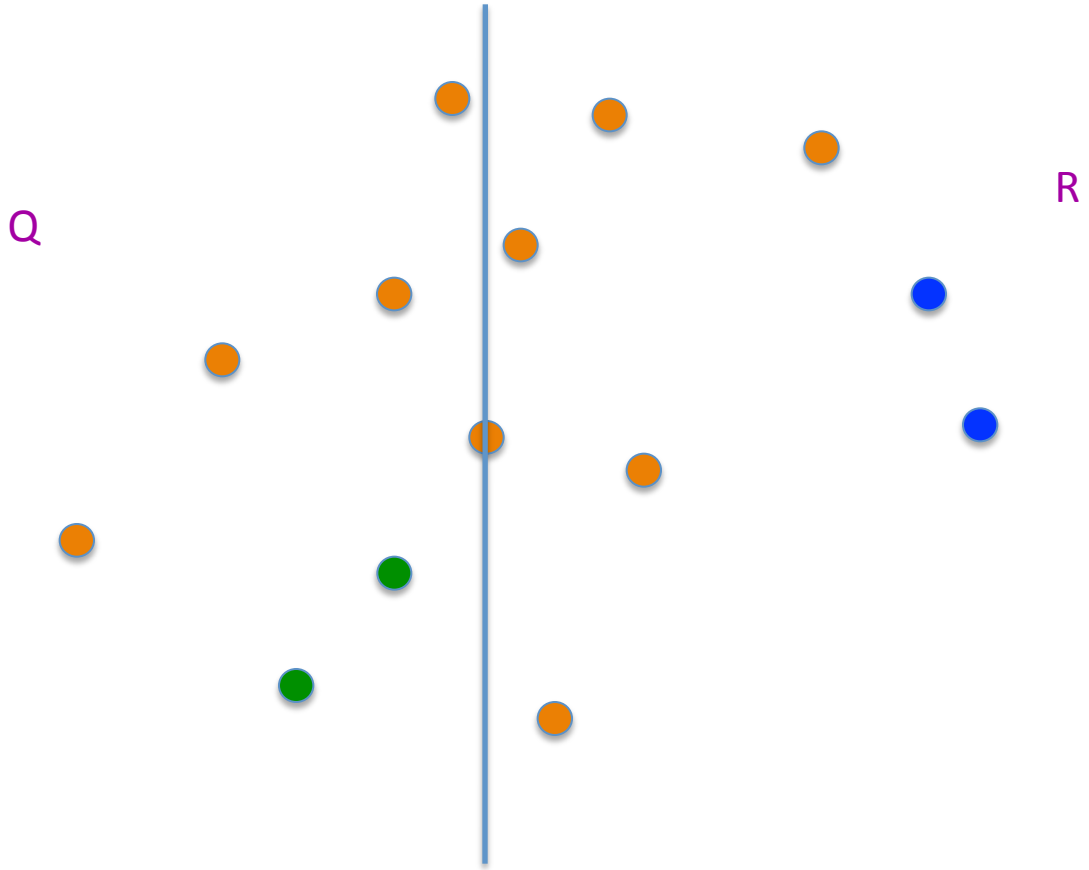
An easy case



All "crossing" pairs have distance > δ

δ = min (**blue**, **green**)

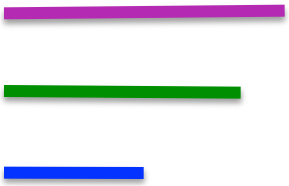
Life is not so easy though

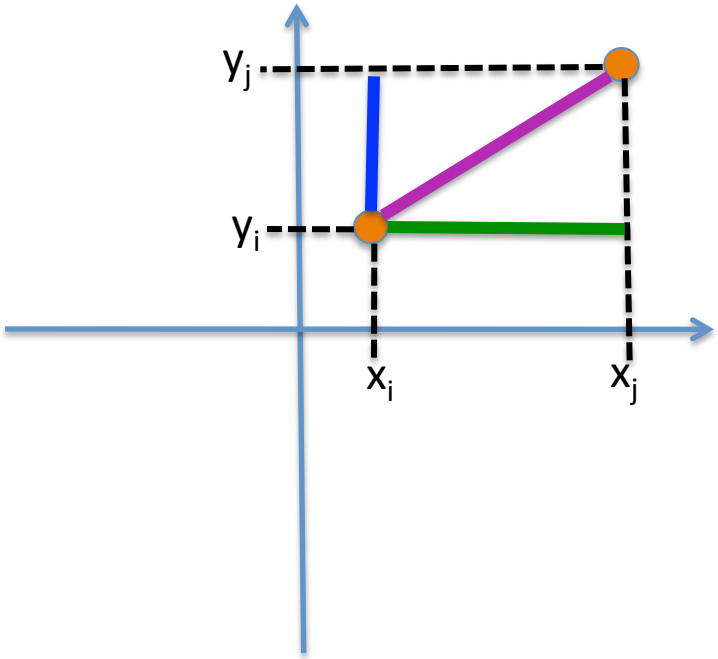


δ = min (**blue**, **green**)

Euclid to the rescue (?)

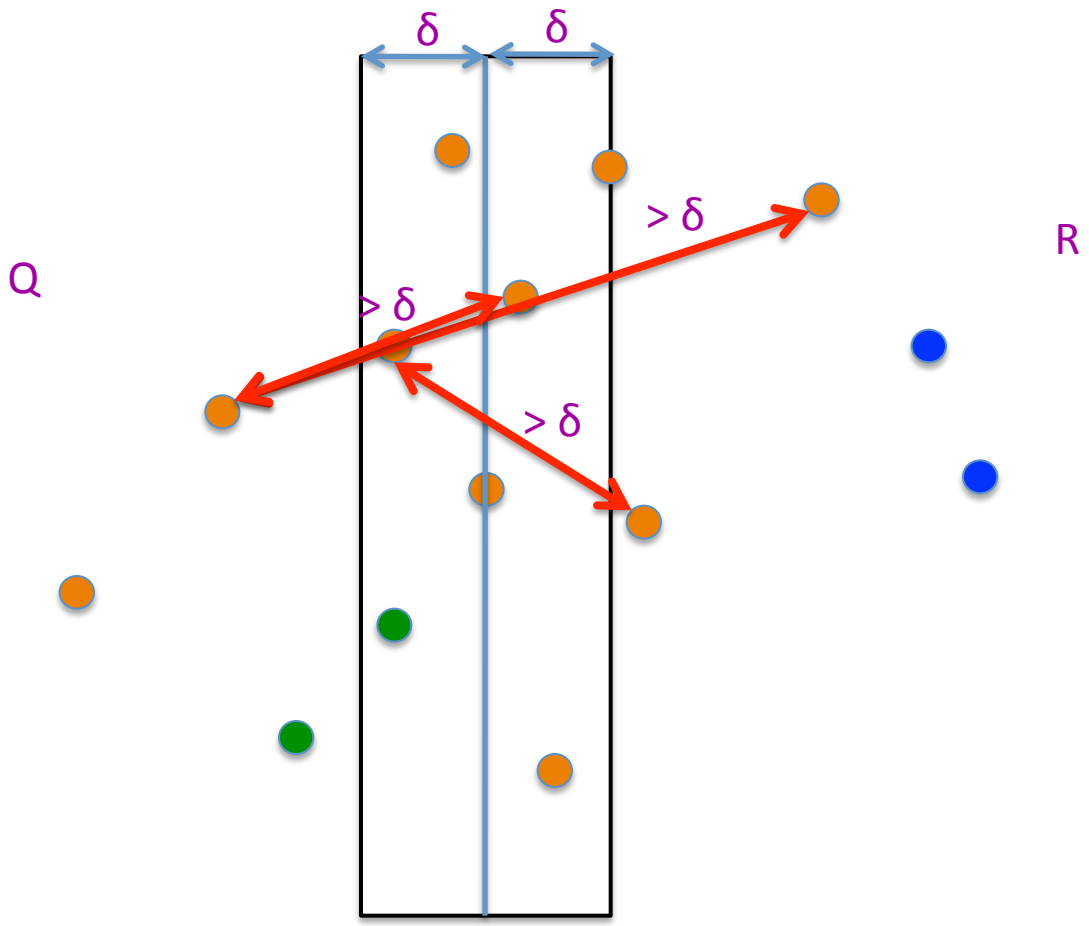
$$d(p_i,p_j) = ((x_i-x_j)^2+(y_i-y_j)^2)^{1/2}$$





The distance is larger than the **x** or **y**-coord difference

Life is not so easy though



δ = min (**blue**, **green**)

All we have to do now

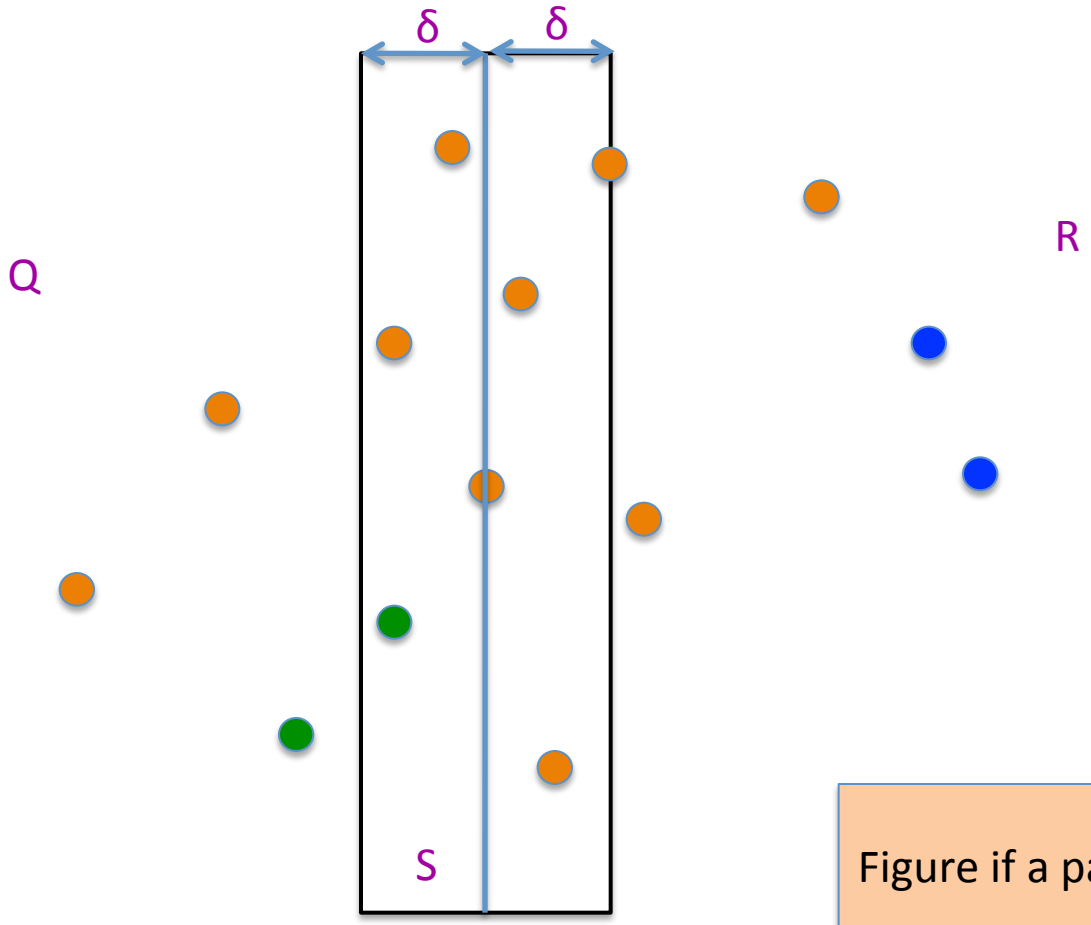


Figure if a pair in S has distance < δ

δ = min (**blue**, **green**)

The algorithm so far…

O(n log n) + T(n)

Input: n 2-D points $P = \{p_1,\ldots,p_n\}$; $p_i=(x_i,y_i)$

Sort P to get P_x and P_y — O(n log n)

Closest-Pair (P_x, P_y)

If $n < 4$ then find closest point by brute-force

Q is first half of P_x and R is the rest — O(n)

Compute Q_x, Q_y, R_x and R_y — O(n)

(q_0,q_1) = Closest-Pair (Q_x, Q_y)

(r_0,r_1) = Closest-Pair (R_x, R_y)

δ = min ($d(q_0,q_1)$, $d(r_0,r_1)$) — O(n)

S = points (x,y) in P s.t. $|x - x^*| < \delta$ — O(n)

return Closest-in-box (S, (q_0,q_1), (r_0,r_1)) — Assume can be done in O(n)

$T(< 4) = c$

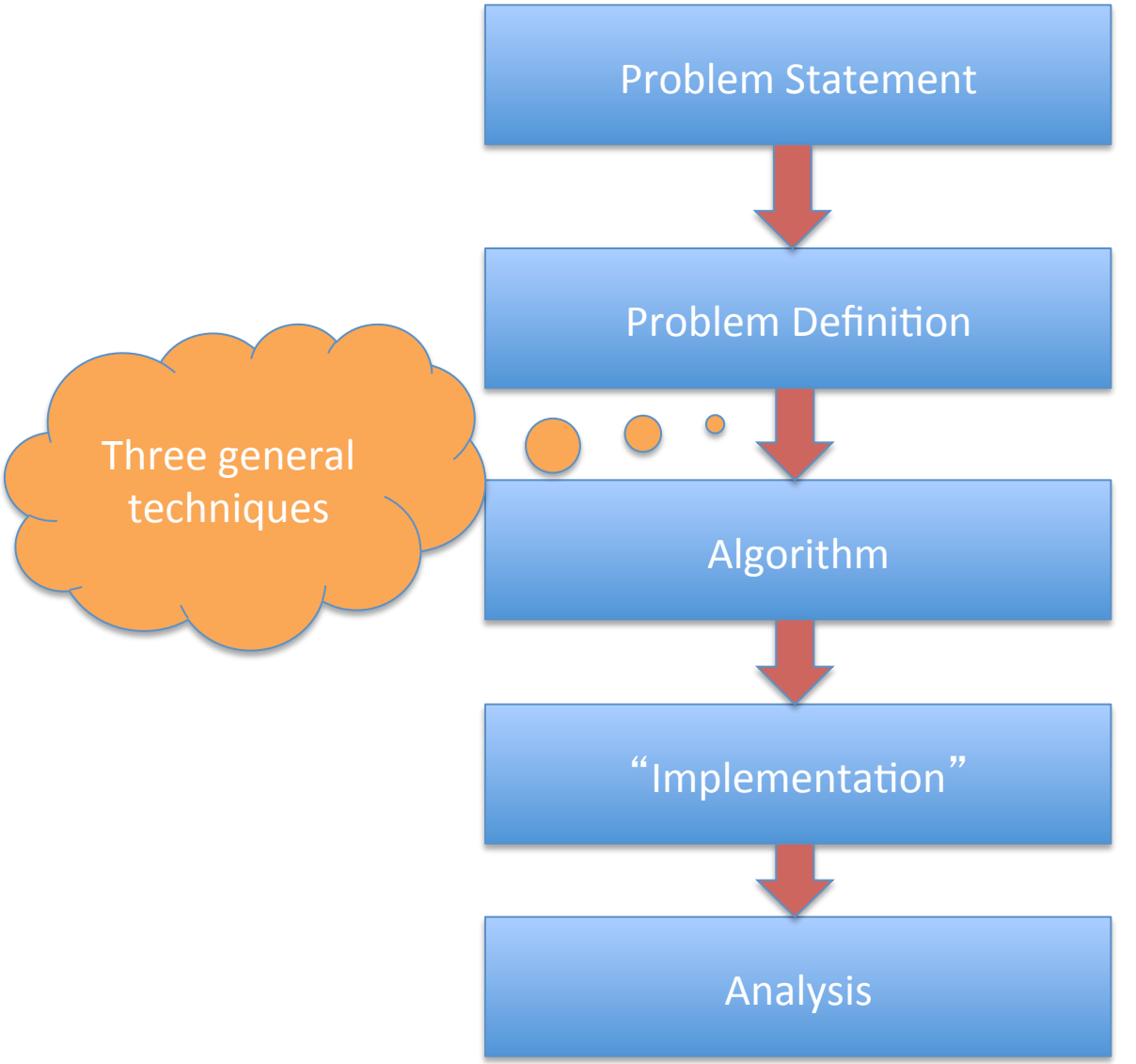
$T(n) = 2T(n/2) + cn$

O(n log n) overall

Rest of today's agenda

Implement Closest-in-box in O(n) time

High level view of CSE 331

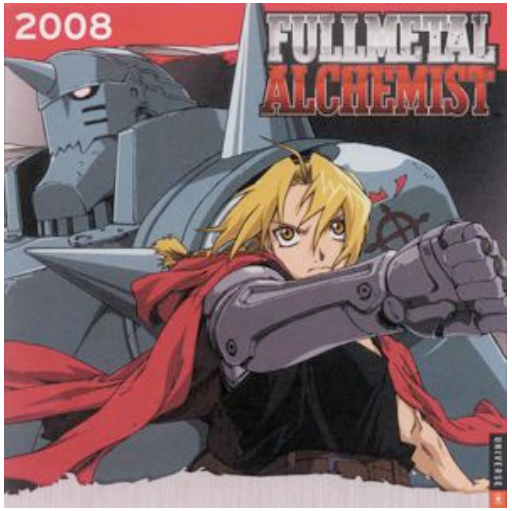


Data Structures

Correctness+Runtime Analysis

Greedy Algorithms

Natural algorithms



Reduced exponential running time to polynomial

Divide and Conquer

Recursive algorithmic paradigm

Reduced large polynomial time to smaller polynomial time

A new algorithmic technique

Dynamic Programming

Dynamic programming vs. Divide & Conquer

Same same because

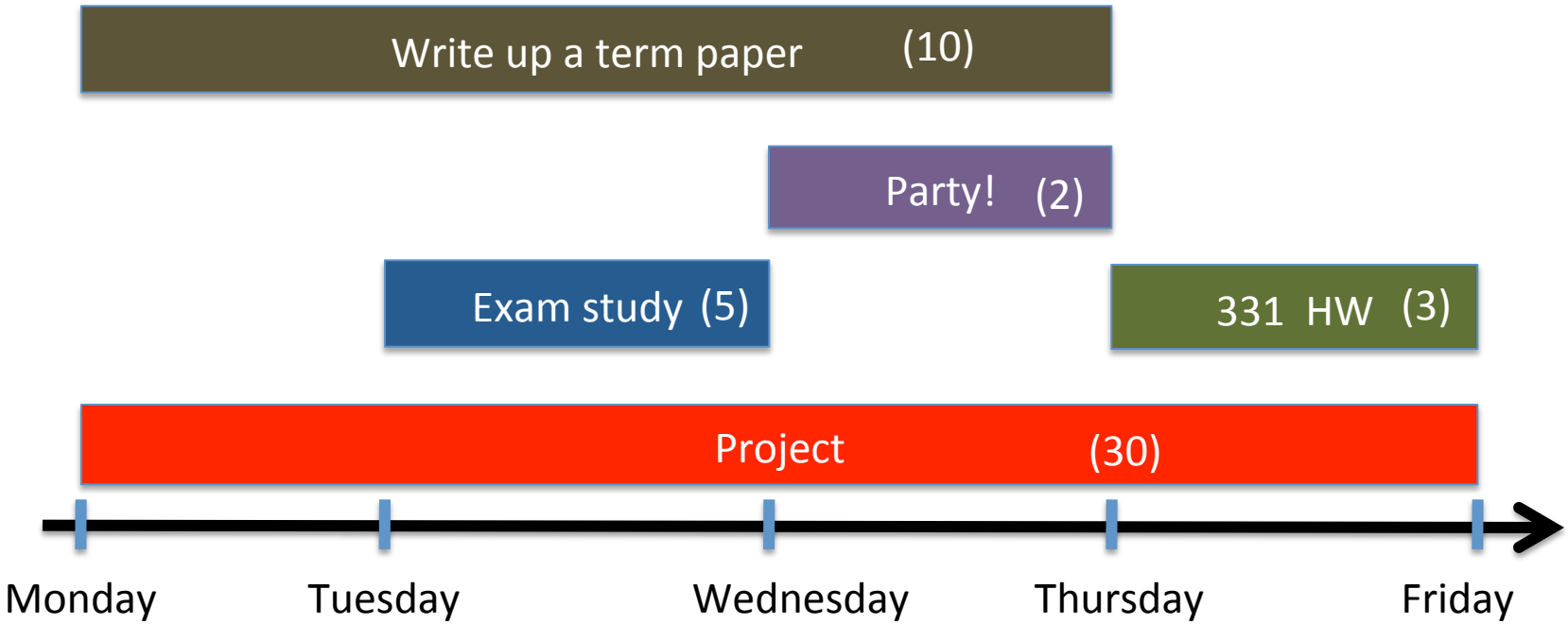
Both design recursive algorithms

Different because

Dynamic programming is smarter about solving recursive sub-problems

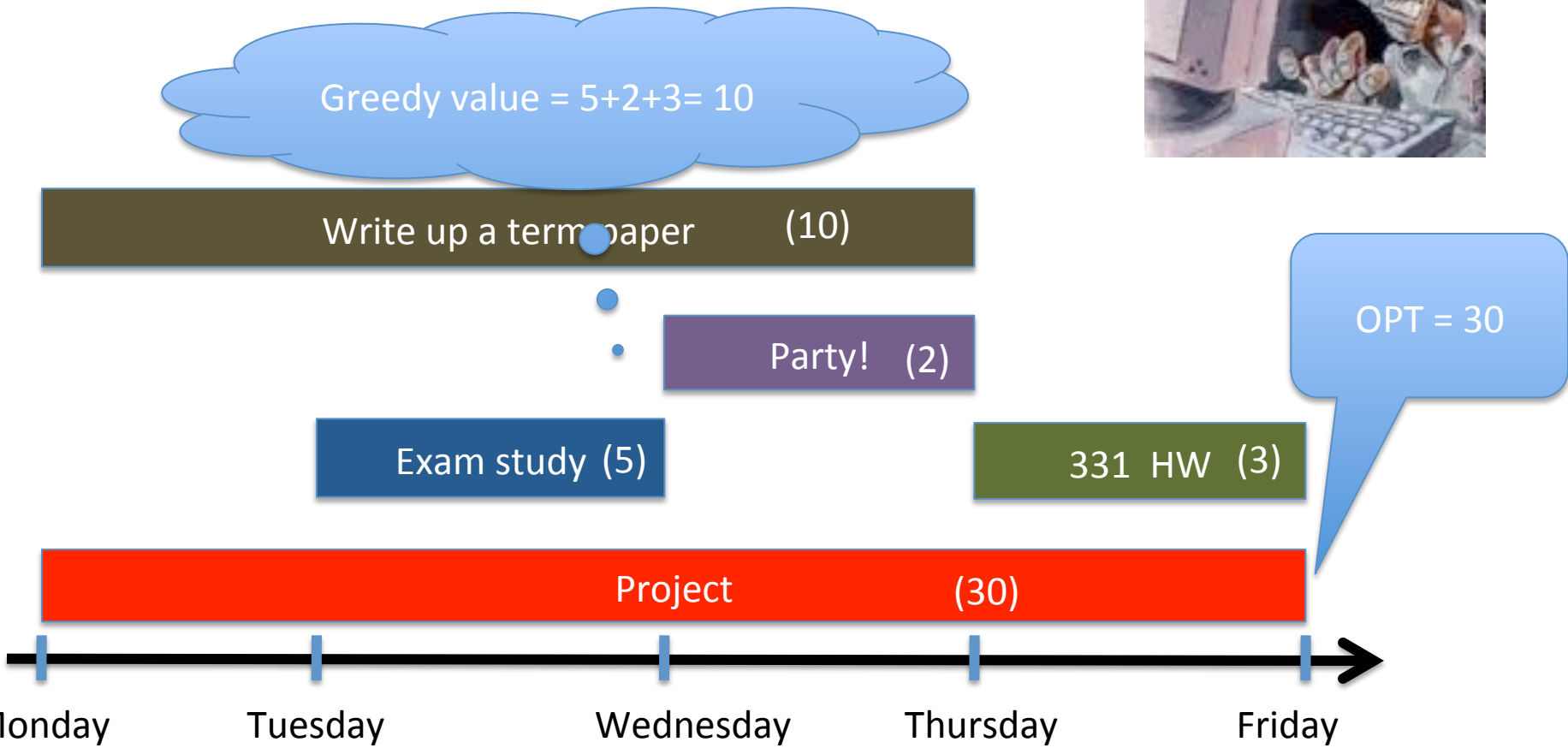
End of Semester blues

Can only do one thing at any day: what is the optimal schedule to obtain maximum value?



Previous Greedy algorithm

Order by end time and pick jobs greedily



Today's agenda

Formal definition of the problem

Start designing a recursive algorithm for the problem