Claim 4: Every $u \in V$ is the head of ≤ 1 edges

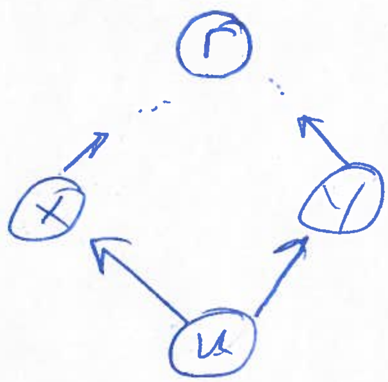
Pf Assume for contradiction that

some $u \in V$ is the head of 2 or more edges

Suppose there are the edges (u,x) and $(u,y) \in E$

$x \neq y$

T:



T is an undirected graph that is connected with no cycles (T is a tree)

but there is a cycle

$u, x, \dots, r, \dots, y, u$

Even if $y = r$, u, x, r, u is a cycle

So there is a cycle in T, which is a contradic

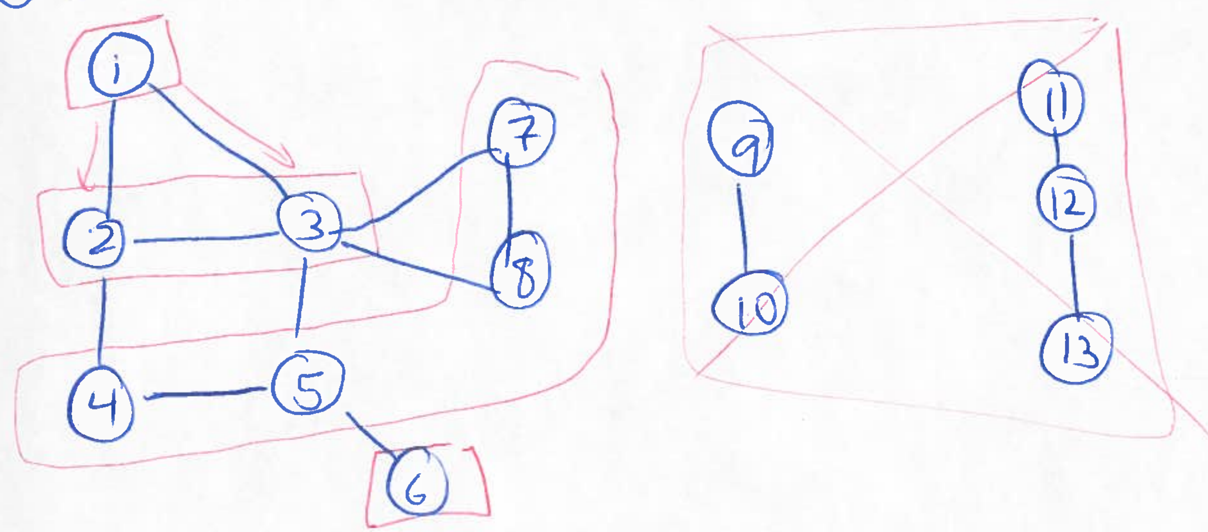
So no $u \in V$ is the head of 2 or more edges $\square$

Input A graph $G = (V, E)$, $s, t \in V$

Output Yes if $\exists$ $s - t$ path

No otherwise

G:



BFS starting at 1

1 $\quad$ L_0

2 3 $\quad$ L_1

4 5 7 8 $\quad$ L_2

6 $\quad$ L_3

If a vertex w doesn't appear in any layer there is no path u- from the starting vertex u

A graph G may have multiple BFS trees with the same starting vertex s.

Prop Let T be a BFS tree of $G = (V, E)$

Let $x \in L_i$, $y \in L_j$, $(x, y) \in E$

Then $|i - j| \leq 1$ (i & j differs by at most