A graph G may have multiple BFS trees with the same starting vertex s.

Prop Let T be a BFS tree of $G=(V,E)$

Let $x \in L_i$, $y \in L_j$, $(x,y) \in E$

Then $|i-j| \leq 1$ (i & j differ by at most 1)

Pf (idea) w.l.o.g. (without loss of generality) assume $i \leq j$

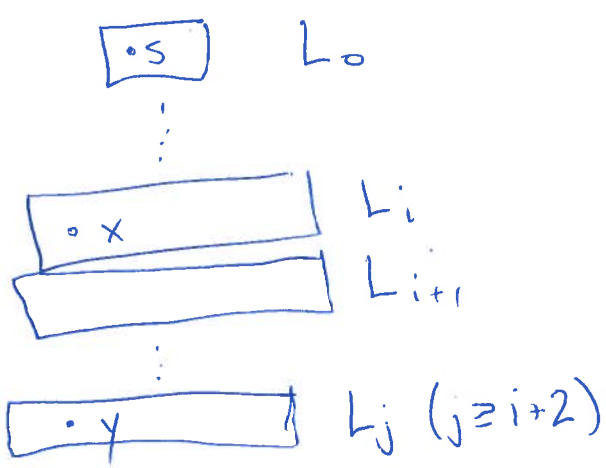
(you can easily swap x and y to get this)

Assume for contradiction that $|i-j| > 1$

$\equiv i < j-1$ (since $i \leq j$)

or $i \leq j-2$

Consider the layers from BFS



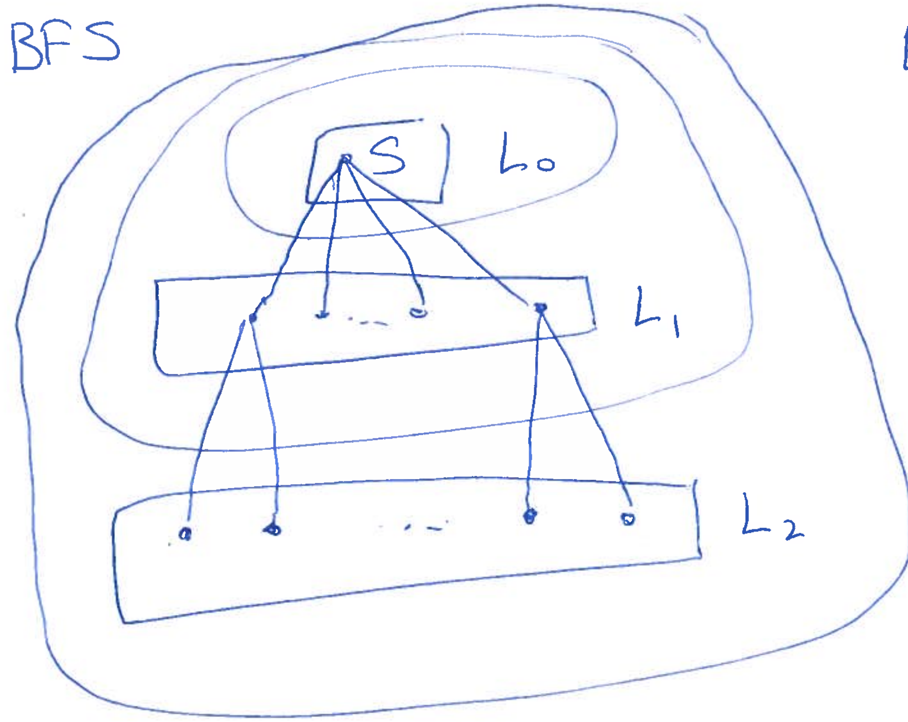
While running BFS algo. to construct L_{i+1},

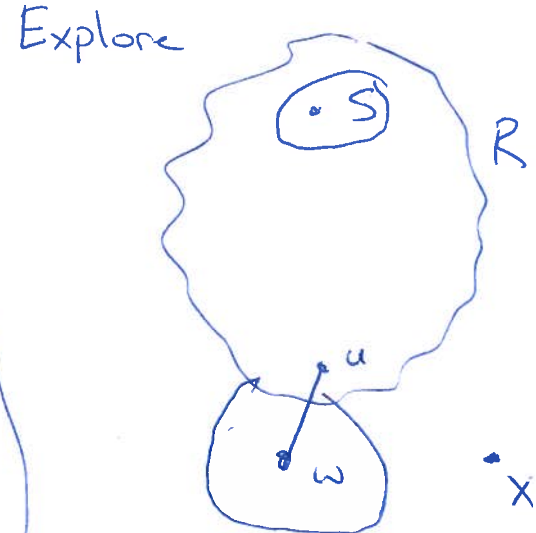
we look in L_i, see $x \in L_i$

we see edge (x,y) and

$y \notin L_0, L_1, \ldots, L_i$, so we place y into L_{i+1}.

This is a contradiction.


BFS
S
L_0
L_1
L_2


Explore
S
R
u
w
x

Explore(s)   (Graph $G=(V,E)$)

1. $R \leftarrow \{s\}$
2. while $(u,w) \in E$ s.t. $u \in R$, $w \notin R$
   Add w to R
3. Return $(R^* \leftarrow R)$

<u>Def</u> The set of all nodes reachable from node s is called the <u>connected component</u> of G containing s. (Can be denoted by $CC(s)$).

<u>Thm</u> R^* = the connected component of G containing s $(R^* = CC(s))$

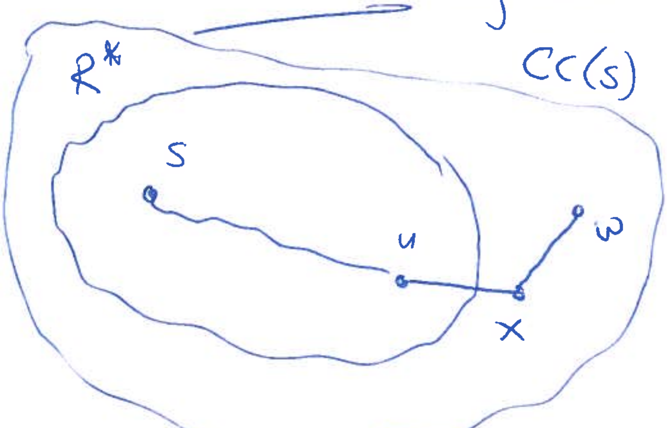
<u>Pf (idea)</u> Argue $R^* \subseteq CC(s)$ and $CC(s) \subseteq R^*$

<u>Lemma1</u> $w \in R^* \Rightarrow w \in CC(s)$

<u>Exercise:</u> Prove by induction on $|R|$ that $w \in R^* \Rightarrow w \in CC(s)$ (in fact $R \subseteq CC(s)$)

<u>Lemma 2:</u> $w \in CC(s) \Rightarrow w \in R^*$

<u>Pf (idea)</u> by contradiction assume $w \in CC(s)$ but $w \notin R^*$



$\Rightarrow \exists$ s-w path but $s \in R^*$, $w \notin R^*$

$\Rightarrow \exists\, u \in R^* \wedge x \notin R^*$ st $(u,x) \in E$

$\Rightarrow$ Explore(s) has not terminated yet.

$\Rightarrow$ Contradiction to @

If this R^* exists $\Rightarrow$ Explore(s) terminated. (@)