Sep 29, 2014

BFS(s)

$O(n)$ $\left\{ \begin{array}{ll} 0.\ cc[s] = T \ \&\ cc[w] = F \ \forall w \neq s & O(n) \\ 1.\ i \leftarrow 0 & O(1) \\ 2.\ L_0 \leftarrow \{s\} & O(1) \end{array} \right.$

3. while $L_i \neq \phi$ ← # itrs: T_1

$L_{i+1} \leftarrow \phi$ ← : $O(1)$

for every $u \in L_i$ ← # itrs: T_2

times algo gets here: T_{12} → for every $(u,w) \in E$ ← # itr: T_3

times algo executes this if = T_{123} → if $(cc[w] = F)$

$cc[w] \leftarrow T$

Add w to L_{i+1}

$O(1)$

$i{+}{+}$; ← $O(1)$

$O(n) + \underbrace{T_1 \times T_2 \times T_3}_{T_{123}} \times O(1)$ | $T_{123} \leq T_1 \times T_2 \times T_3$

Run time: $O(n) + O(T_{123})$

$O(n^3)$: # layers $\leq n \Rightarrow T_1 \leq n$.

each layer has $\leq n$ vertices $\Rightarrow T_2 \leq n$

neighbors of $u \leq n \Rightarrow T_3 \leq n$

$\Rightarrow T_{123} \leq O(n^3)$

$\Rightarrow$ overall $O(n) + O(n^3) = O(n^3)$

$O(n^2)$: $T_{12} \leq O(n^2)$ [$T_{12} \leq T_1 \times T_2 \leq O(n^2)$

Since each layer is disjoint $\Rightarrow T_{12} \leq n$

$T_{123} \leq T_{12} \cdot T_3 \leq O(n \cdot n) = O(n^2)$

Overall $O(n) + O(n^2) = O(n^2)$

$\Theta(m+n)$: $T_{123} \leq \sum_{u \in V} n_u = 2m$

Overall : $O(n) + O(m) = \boxed{O(m+n)}$

input size $N = \Theta(m+n)$

$\Rightarrow$ BFS is a linear time algo!