Oct 8, 2014

$A^* = \{i_1, \ldots, i_k\}$ $\quad k \le m$

$O = \{j_1, \ldots, j_m\}$

Lemma: $\forall \; 1 \le r \le k; \; f(i_r) \le f(j_r)$

Pf: By induction on r

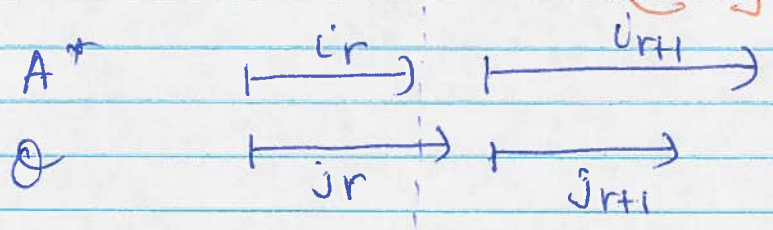
Base case: $r = 1$ ✓ (as $f(i_1)$ is smallest finish time)

Assume true upto some $r \ge 1$.

(Argue for $r+1$)

for contradiction assume $f(i_{r+1}) > f(j_{r+1})$

(Note: $f(i_r) \le f(j_r)$) ← by inductive hypothesis



end of r^{th} iteration of algo $\Rightarrow$ $j_{r+1} \in R$ at that stage

$\Rightarrow$ Algo cannot have picked i_{r+1} as

(1) $j_{r+1} \in R$ and (2) $f(j_{r+1}) < f(i_{r+1})$

$\Rightarrow$ contradiction ⊗

$O(n^2)$

1. $R \leftarrow [n]$ $\quad O(n)$
2. $A \leftarrow \phi$ $\quad O(1)$
3. While $R \neq \phi$ ← $\le n$ iteration
   - $O(n)$ → (3.1) Let $i \in R$ be interval w/ smallest $f(i)$
   - $O(1)$ → (3.2) Add i to A
   - $O(n)$ → (3.3) Remove all $j \in R$ that conflict with i
4. Return A. ← $O(1)$

(Scan R) $\Rightarrow O(n^2)$ overall

If R is sorted $O(n)$ possible

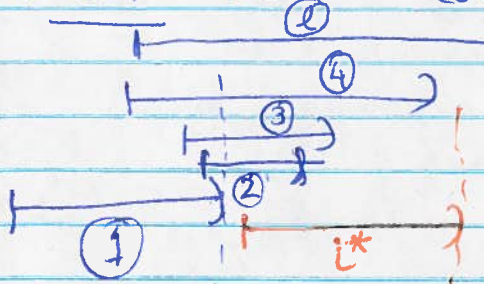
$O(n)$ time.

Assume: Init R is sorted by finish time ($O(n \log n)$)

→ Assume if $i < j$ $f(i) \leq f(j)$

→ Compute $s[i]$ → start time of i'th guy

IDEA: Remove conflicting j's as needed



Check 2 & see if $s[2] < f(1)$

Continue with all $i > 1$ s.t $s[i] < f(1)$

Let $i^* > 1$ be smallest $i > 1$ s.t $s[i] \geq f(1$

Add i^* to A