Oct 13, 2015

Def: Given a schedule S a pair (i,j) is an inversion in S if (1) i is scheduled before j AND (2) $d_i > d_j$.

Def: $\#inv(S) \rightarrow$ # inversions in S.

Q: what is max #inv? $\leq n^2$.

Lemma 1: Any two schedules (for same input) both with 0 idle time & 0 # inversions has the same max. lateness

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Lemma 2: $\exists$ an optimal schedule O with 0 idle time & $\#inv(O) = 0$.

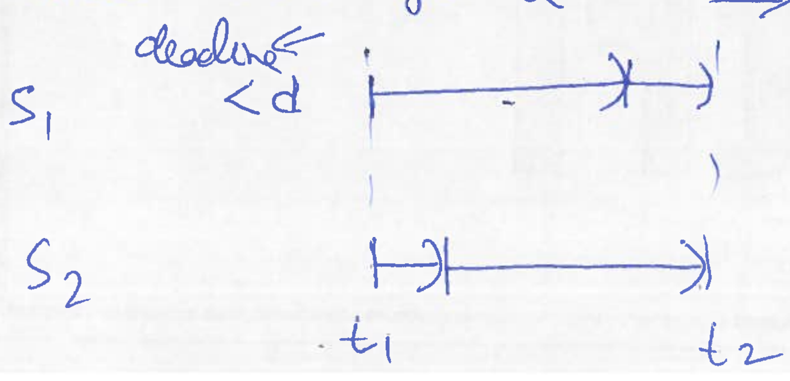
Obs 1: A has 0 idle time & $\#inv(A) = 0$

Obs 2: $\exists$ an O with 0 idle time

Pf of Lemma 1: S_1 & S_2 be schedules for same input both w/ 0 idle time & 0 #inv.

Claim: for every distinct value of deadline d all jobs with $d_i = d$ are scheduled in the same time range

$d \rightarrow$ deadline $> d$



Assume Claim is true
max lateness of jobs w/ $d_i = d$
$= \max(t_2 - d, 0)$

Pf Idea for Claim:

$\# inv = 0$ $\quad \leq d \longleftarrow \; d \; \longrightarrow \geq d$

$\Rightarrow$ all jobs with the same deadline are scheduled together.

$+$ $\quad \overset{d}{\longmapsto} \;\; \cancel{\overset{d'}{\longmapsto}} \;\; \overset{d}{\longmapsto} \qquad d' \neq d$

$idle = 0 \Rightarrow$ all jobs with deadline d have the same time range in S_1 & S_2.
(can argue by induction on value of deadlines).

Pf of Lemma 2: Use an exchange argument.

$Q_1 \longrightarrow Q_2 \quad \cdot \quad \longrightarrow Q_i \longrightarrow Q_{i+1}$

0 idle time

$\cdots \longrightarrow Q_m = Q$

idle $Q = 0$
$\# inv(Q) = 0$
$L(Q) = L(Q_1)$

$$\boxed{\begin{array}{l} \text{If } \# inv(Q_i) > 0 \\ \# inv(Q_{i+1}) < \# inv(Q_i) \end{array}}$$

idle $(Q_{i+1}) = 0$

$\longleftarrow L(Q_{i+1}) \leq L(Q_i)$

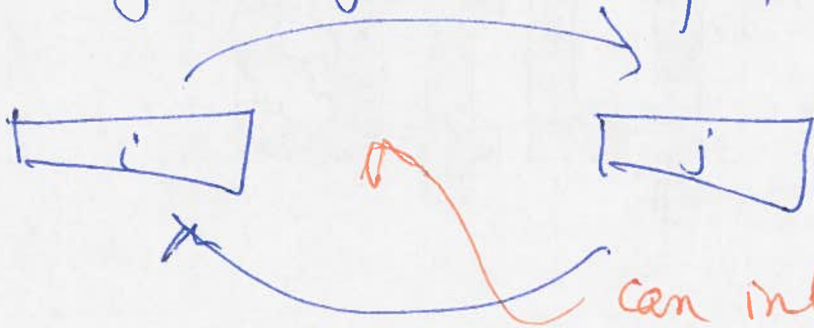
$\cancel{L(Q)}$ $L(Q_m) \leq L(Q_{m-1}) \leq \cdots \leq L(Q_2) \leq L(Q_1)$

Q''

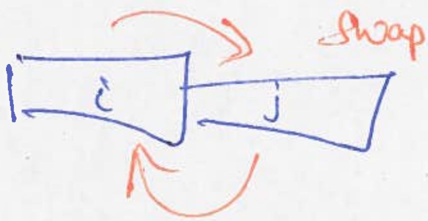
$\Rightarrow L(Q_m) \leq L(Q_1)$

Say (i, j) is an inversion.

→ best way to get rid of it is to swap



can introduce new inv



(i, j) inv