Oct 15, 2014

Lemma: We can convert an optimal solution Θ' with 0 idle time to another optimal solution Θ with 0 idle time & 0 # inversions

---

Pf: $\quad$ idle $(\Theta_1) = 0$

$$\Theta' = \Theta_1 \to \Theta_2 \to \cdots \to \Theta_i \to \Theta_{i+1} \to \Theta_m = \Theta$$

Properties: $\forall i$

(i) idle $(\Theta_{i+1}) = 0$

(ii) If $\#inv(\Theta_i) > 0 \Rightarrow \#inv(\Theta_{i+1}) = \#inv(\Theta_i) - 1$

(iii) $L(\Theta_{i+1}) \leq L(\Theta_i)$ $\xrightarrow{i=m-1}$ $L(\Theta_m) \leq L(\Theta_{m-1}) \leq L(\Theta_{m-2}) \leq L(\Theta_{m-3}) \cdots \leq L(\Theta_2) \leq L(\Theta_1)$

$\forall i \Rightarrow L(\Theta_m) \leq L(\Theta_1) = L(\Theta')$

$L(\Theta_m) = L(\Theta)$

$\Rightarrow L(\Theta) = L(\Theta')$ (hence, opt.)

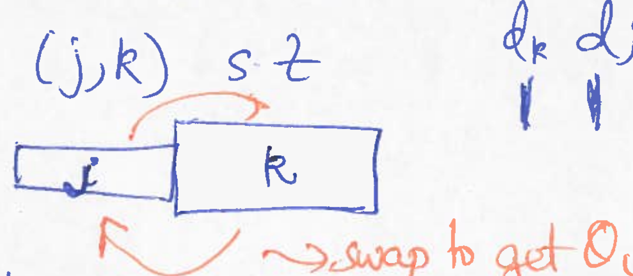
as Θ' is optimal

Conversion: $\Theta_i \to \Theta_{i+1}$ $\quad \#inv(\Theta_i) > 0$

~~idle~~ idle $(\Theta_i) = 0$

Property (a) (Since $\#inv(\Theta_i) > 0$) $\exists$ (j,k) s.t.

i) (j,k) is an inversion (ii)



Property (b) Swap j & k to get Θ_{i+1} $\quad$ ⤳ swap to get Θ_{i+1}

$\#inv(\Theta_{i+1}) = \#inv(\Theta_i) - 1$

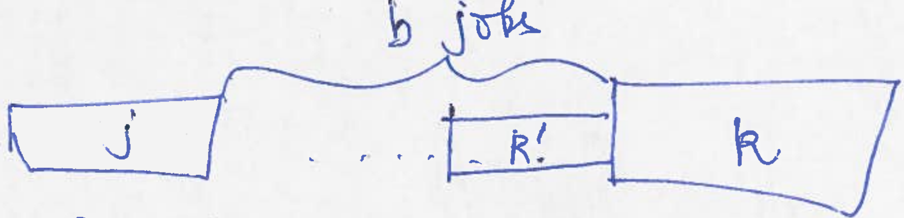
idle $(\Theta_{i+1}) = 0$

} follows by construction of Θ_{i+1}

Property (c): $L(\Theta_{i+1}) \leq L(\Theta_i)$

Pf of property (a)

Know:

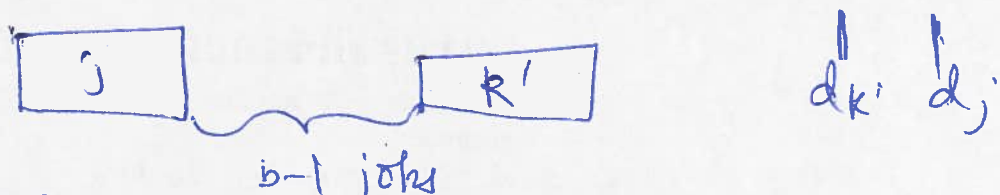


Case 1: If j & k are right next to each other: ✓

Case 2: k' next to k closer to j

Case 2.1: $d_{k'} > d_k \Rightarrow (k', k)$ is our pair.

Case 2.2 $d_{k'} \le d_k < d_j \Rightarrow d_{k'} < d_j$

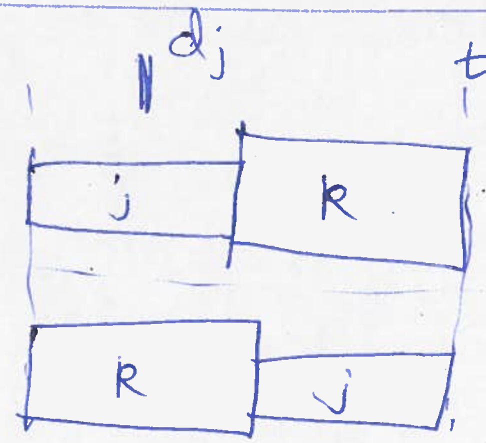


Continue till $b = 0 \Rightarrow$ Case 1 ✓

Property (c)

Q_i

same



same

Q_{i+1}

$L(Q_{i+1}) \le L(Q_i)$

$\forall\, m \notin \{j, k\}$ $\quad \ell(m) = \ell'(m)$

lateness in Q_i lateness in Q_{i+1}

$\Rightarrow$ Enough to argue $\max\{\ell'(j), \ell'(k)\} \le \max(\ell(j), \ell(k))$

$\max\{\ell'(j), \ell'(k)\} \le \ell(k)$

(i) $\ell'(k) \le \ell(k)$ (as in Q_{i+1} k finishes $< t$)

~~Now if we could pr~~

→ Possible that $l'(j) > l(j)$

(ii) $l'(j) = t - d_j$ (assume $d_j < t$)

$\leq t - d_k$ $\quad d_j > d_k$ as (j,k) an inv

$= l(k)$ $\quad \Rightarrow -d_j < -d_k$

$\blacksquare$