Oct 17 Shortest Path Problem

Input: Directed $G = (V, E)$

$\forall e \in E, \quad l_e \geq 0$

$s \in V$

Output: $\forall\ t \in V$, output shortest s-t path

$l(P) = \sum_{e \in P} l_e$

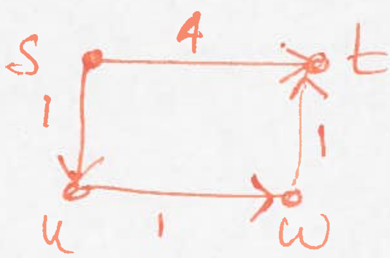
$\boxed{d(t) \rightarrow \text{length of the shortest } s\text{-}t \text{ path.}}$

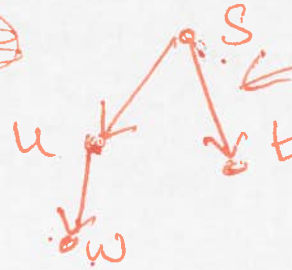
Case 1: $\forall e \in E, \ l_e = 1$

Use BFS starting at s (BFS tree rooted at s gives the shortest path)

Case 2: general lengths

Idea 1: Run BFS by ignoring edge length





not shortest path

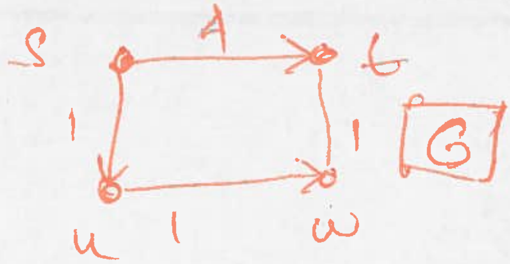
Q: Reduce G with arbit l_e $\longrightarrow$ G' with all $l'_e = 1$

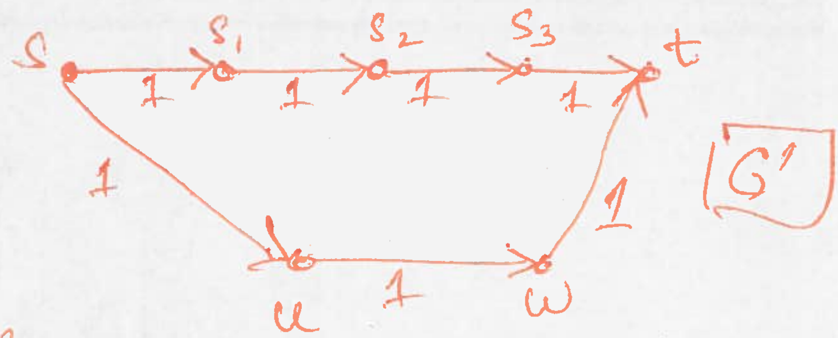
$\boxed{\text{all integers}}$

use BFS

shortest paths $\longleftarrow$ shortest paths

Reduce: $u \xrightarrow{l_e} w$ $\qquad$ $u \underbrace{\circ \xrightarrow{1} \circ \xrightarrow{1} \circ \xrightarrow{1} \circ \cdots \xrightarrow{1} \circ}_{l_e} w$

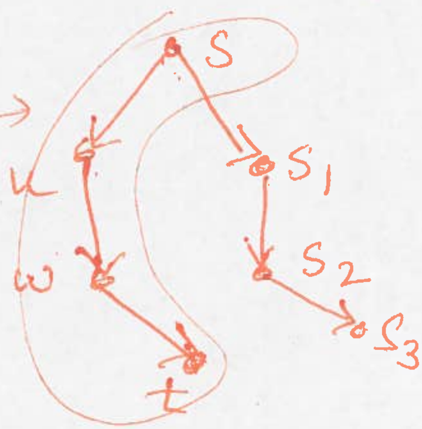




1-to-1 correspondence b/w shortest paths in G & G'

BFS on G':

shortest paths



G' has n' vertices & m' edges

$\leftarrow O(n'+m')$

$l_{max} = \max_e l_e$

$n' \leq l_{max}(n+m)$

$m' \leq l_{max} m$

$\Rightarrow$ overall runtime $O(l_{max}(n+m))$

$\rightarrow$ i/p size $\Theta(\log l_{max}(m+n))$

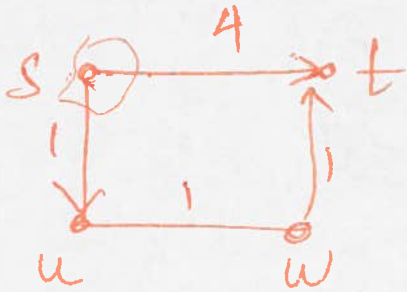
$l_{max} = n^{100}$

Want: $l_{max} = n^{O(1)} \Rightarrow$ # input registers needed $= \Theta(n+m)$

RAM model

Want: $O(n+m)$ algo for

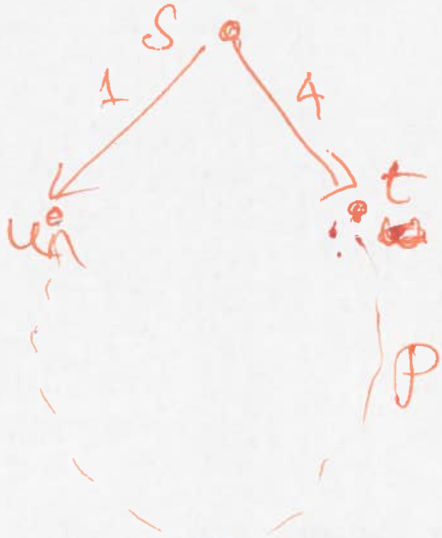
Toward's Dijkstra's algo (greedy algo)



$d(s) = 0$

say $d(u) = 1$

$d(w) = 1$ is correct. $\Leftarrow$

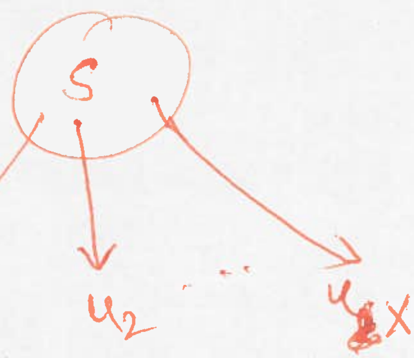


alternate path ~~thr~~ through P^w

$l(P) = 4 + \underbrace{\text{stuf}}_{\geq 0}$
≥ 4

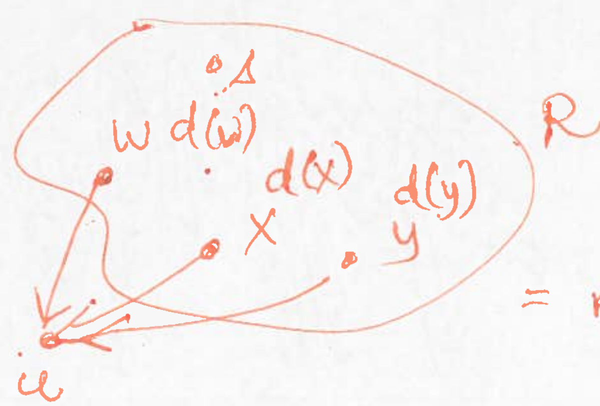
as +ve, len

First step



Pick u_i s.t. l_{s,u_i} is smallest.

General step



$u \notin R$

Upperbound on $d(u)$;

$d'(u) = \underset{\min}{\cancel{\max}} \{ d(w) + l_{w,u},\ d(x) + l_{x,u},\ d(y) + l_{y,u} \}$

$$d'(u) = \min_{\substack{w \in R \\ e=(w,u) \in E}} \{ d(w) + l_e \}$$