Nov 12, 2014

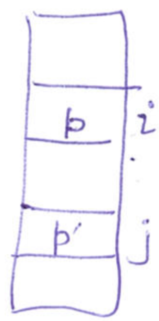
KICKASS PROPERTY LEMMA

~~If~~ $\forall$ $p \neq p' \in S$ s.t. $d(p,p') < \delta$,

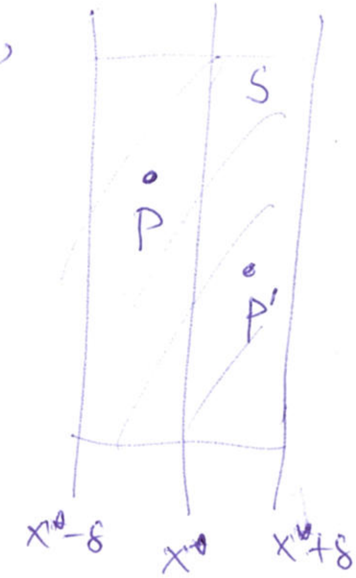
if $S_y[i] = p$

$S_y[j] = p'$

$\Rightarrow |i-j| \leq 15$ (!)



S_y



Algorithmic Consequence:

Want: Algo computes closest pair of crossing pts $p(q, r)$ $q, r \in S$ assuming $d(q,r) < \delta$.

$q \in Q$ $r \in R$

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for $i = 1, \ldots, n'$ $\qquad |S_y| = n'$

find the closest pair of points amongst $(S_y[i], S_y[i+1]), (S_y[i], S[i+2]), \ldots$

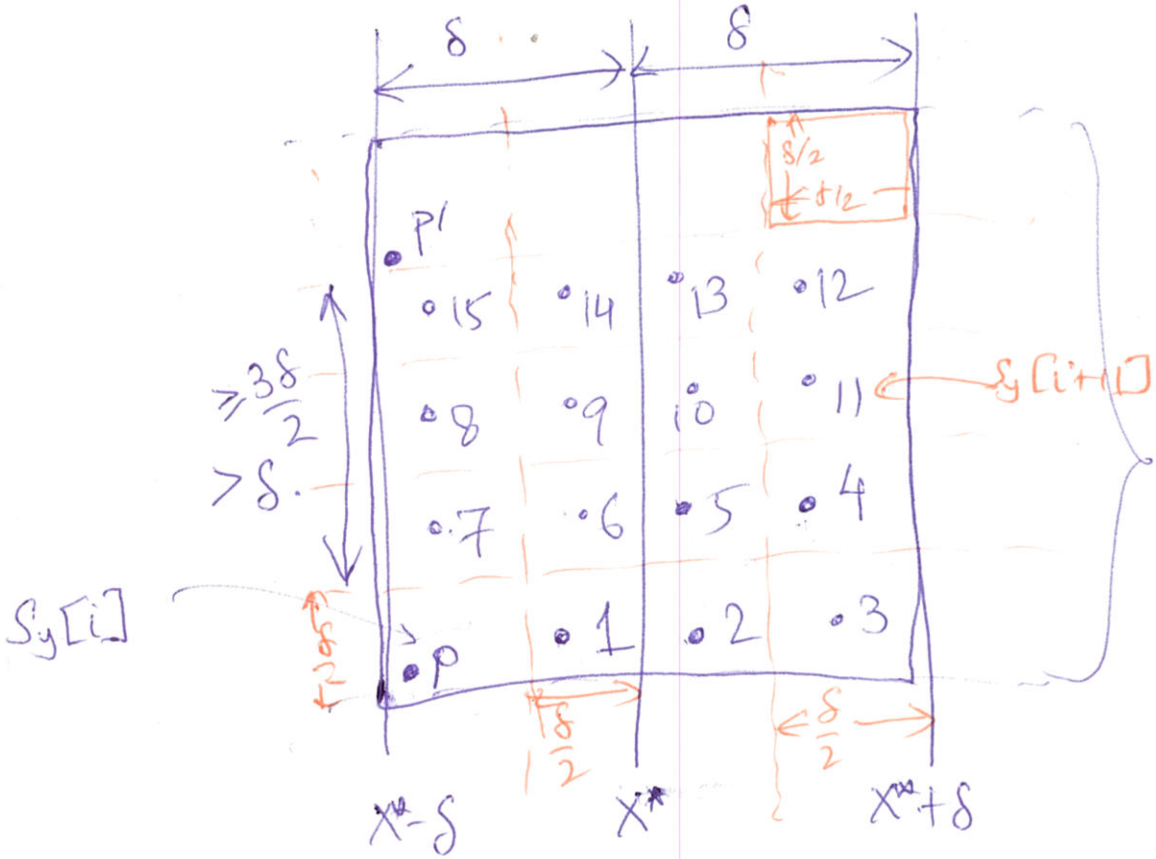
(q_i, r_i) $\qquad \ldots, (S_y[i], S[i+15])$

$\rightarrow$ Compute closest pair of pts among $(q_1, r_1), \ldots, (q_{n'}, r_{n'})$ $\quad (q^*, r^*)$

$\rightarrow$ check if $d(q^*, r^*) < \delta$

Correctness: Kickass Property Lemma.

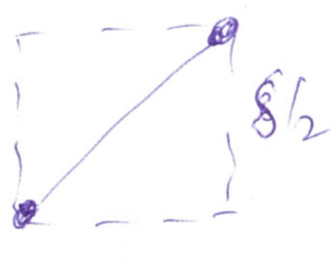
Pf of Kickass Property Lemma



p, p'
$d(p,p') < \delta$
(Assum p & p' > 15 apart.)

Claim: No $\frac{\delta}{2} \times \frac{\delta}{2}$ box has > 1 pt in it.

Pf:



either in Q or R
2 pts in box ∈ Q or ∈ R
Farthest apart pts

$\leq \sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2} = \sqrt{\frac{\delta^2}{2}}$

$= \frac{\delta}{\sqrt{2}} < \delta$

not possible
as every pair of pts
in Q ≥ δ apart
in R ≥ δ apart