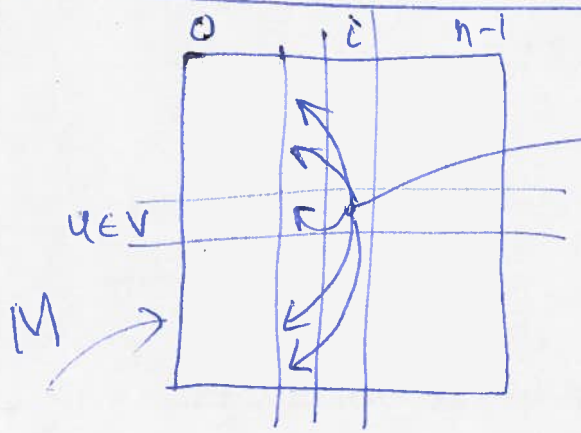
Nov 24, 2014

$$OPT(u,i) = \min\left\{ OPT(u,i-1),\ \min_{(u,w)\in E} c_{u,w} + OPT(w,i-1) \right\}$$



$OPT(u,i)$

Order: compute values column by column (start $i=0,1,\ldots$)

Bellman-Ford Algo

(G: adjacency list format)

0. $M[t,0] = 0$, $M[u,0] = \infty$ $\forall u \neq t$ $[O(n)]$
1. For $i = 1,\ldots,n-1$ $[\leq n]$
   For $u \in V$ $[\leq n]$ $\quad O(n_u)$
   $M[u,i] \leftarrow \min\{ M[u,i-1],$ $[\leftarrow O(n)]$
   $\min_{(u,w)\in E} c_{u,w} + M[w,i-1] \}$

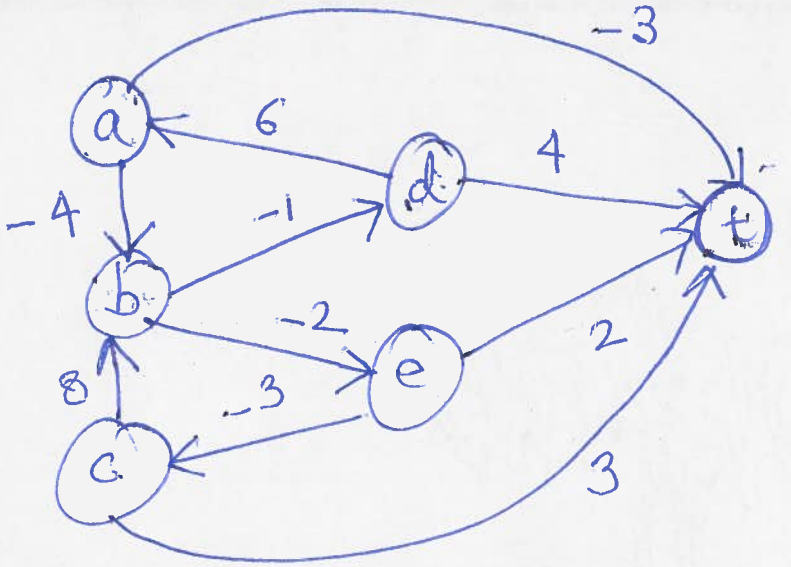
   $O(n^3)$
2. Output $M[s,n-1]$ $\forall s$. $[O(n)]$

$O(n \leq \sum_{u\in V} n_u = m)$

→ Overall: $O(n^3) \to O(mn)$ [by noticing min takes $O(n_u)$ time]

Space: $O(n^2) \to O(n)$ ← just keep track of last 2 columns.

$n = 6$



	0	1	2	3	4	5
a	∞	-3				
b	∞	∞				
c	∞	3				
d	∞	4	3			
e	∞	2				
t	0	0				

output

$M[a,1] = \min\{\infty, \min\{-3+0, -4+\infty\}\} = -3$

$M[b,1] = \min\{\infty, \min\{-1+\infty, -2+\infty\}\} = \infty$

$M[c,1] = \min\{\infty, \min\{3+0, 8+\infty\}\} = 3$

$M[d,1] = \min\{\infty, \min\{4+0, 6+\infty\}\} = 4$

$M[e,1] = \min\{\infty, \min\{2+0, -3+\infty\}\} = 2$

$M[t,1] = \min\{0\} = 0$

$M[t,i] = 0 \quad (i = 0, \ldots, 5)$

~~$M[t,2] = \min$~~ $M[d,2] = \min\{4, \min\{4+0, 6+3\}\}$

$= 3$

Fix a col i

→ Time for col i $= \sum_{u \in V} O(n_u) = O\left(\sum_u n_u\right)$

$= O(m)$