Sep 3, 2014

Stable Matching / Marriage Problem (NOT feminist)

→ n men $\quad M = \{m_1, \ldots, m_n\}$

→ n women $\quad W = \{w_1, \ldots, w_n\}$

$\boxed{n=2}$

$M = \{BP, BBT\}$

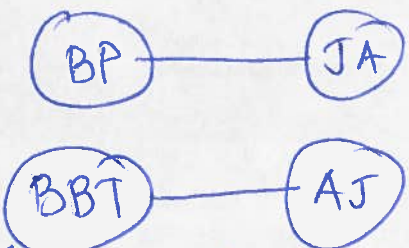
$W = \{JA, AJ\}$

Assignment / Matching

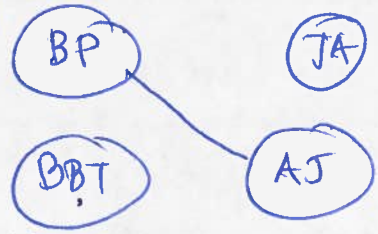
$S \subseteq M \times W \left(\stackrel{def}{=} \{(m,w) \mid m \in M, w \in W\}\right)$

s.t. $\forall m \in M$, $\exists \le$ one $w \in W$ s.t. $(m,w) \in S$

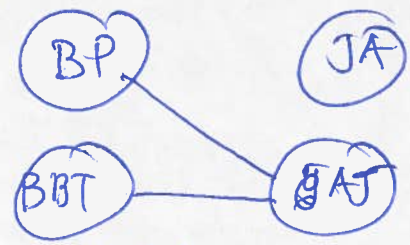
$\forall w \in W$, $\exists \le$ one $m \in M$ s.t. $(m,w) \in S$



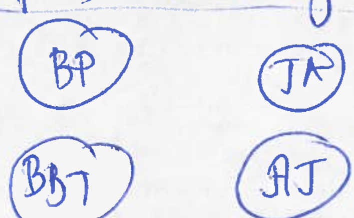
(Perfect) Matching ✓



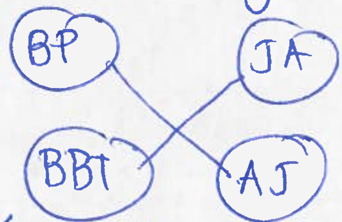
(~~Perfect~~) Matching ✓



Matching ✗



(~~Perfect~~) Matching ✓



(Perfect) Matching ✓

Perfect Matching

In matching def replace "$\le$ one" by "exactly one".

Preference (list)

$\forall m \in M$, list L_m → total ranking of <u>all</u> n women.

$\forall w \in W$, list L_w → __________ men

eg: L_{BP} : $AJ > JA$ $\quad$ L_{JA} : $BP > BBT$

L_{BBT} : $AJ > JA$ $\quad$ L_{AJ} : $BP > BBT$

Def: A stable matching is (1) perfect match (2) has no instability

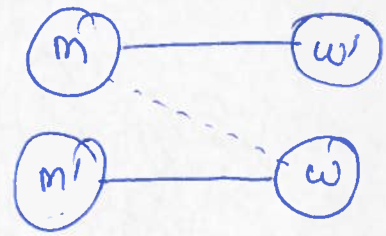
Def: (Given all the pref. lists $&) a perfect matching S a pair $(m,w) \notin S$ if

(1) $m > m'$ in L_w

$\boxed{\text{AND}}$

(2) $w > w'$ in L_m.

$\left[\neg(A \wedge B) \equiv \neg A \vee \neg B\right]$

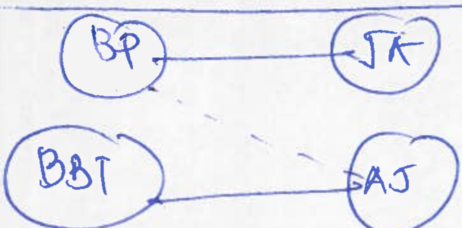


Equivalent def: A perfect matching S is a stable matching s.t. $\forall (m,w) \notin S$ $\left[\begin{array}{l}(m,w') \in S \\ (m',w) \in S\end{array}\right]$

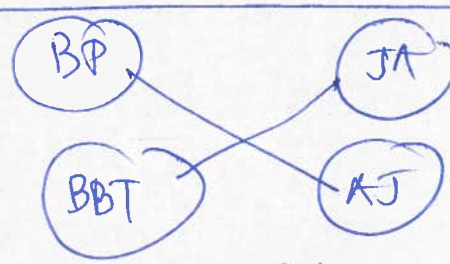
either (i) $m' > m$ in L_w

OR

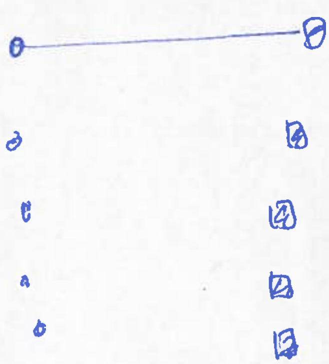
(ii) $w' > w$ in L_m



X stable matching



Stable Matching ✓



Problem:

Input: $L_m \ \forall m \in M$

$L_w \ \forall w \in W$

Output: A stable matching