

Sep 8, 2014

THEOREM: For every instance, the GS algo outputs a stable matching.



COROLLARY: For every instance,  $\exists$  a stable matching.

LEMMA 1: On every input, the GS algo terminates  $\leq n^2$  iterations.

LEMMA 2: The output  $S$  of GS is a perfect matching.

LEMMA 3:  $S$  does not have any instability.

LEMMAS 1+2+3  $\Rightarrow$  THEOREM.

Pf of Lemma 1:

Proof idea: Define progress measure  $P(t)$  at the end of iteration  $t \geq 1$ .

①  $\forall t \geq 1, P(t) \geq 1$ .

②  $\forall t, P(t+1) \geq P(t) + 1$

③  $\forall t \geq 1, P(t) \leq n^2$

$\Rightarrow$  ①, ② & ③  $\Rightarrow$  # iterations  $\leq n^2$ .

integer

$P(t) = t$	$P(t)$ =# engaged pairs
① ✓	① ✓
② ✓	② X
③ X	③ ✓

Pf details:  $P(t) = \overset{\text{total}}{\#}$  proposals made in algo by the end of iteration  $t$ .

①  $P(1) = 1$  (by algo definition)  $\Rightarrow \forall t \geq 1, P(t) \geq P(1) = 1$ .

②  $P(t+1) = P(t) + 1$  (as free woman proposes to a man she has not proposed to yet.)

③  $P(t) \leq |M \times W| = n^2$ .