Sep 10, 2014 | LEMMA 2: S is a perfect matching.

Obs 0: S is a matching.

Obs 1: A man m keeps getting engaged to ~~better~~ more preferred women.

Obs 2: If w proposes to m' after m, $m > m'$ in L_w

Pf of Lemma 2:

LEMMA 4: If a woman w is free $\Rightarrow \exists$ $\geq$ one man m that she has not proposed to yet.

Pf idea: Proof by contradiction (use Lemma 4 & definition of the GS algo).

Pf details: Assume S is not a perfect matching

$\Rightarrow$ $\exists$ a free woman w (at the end of algo).

by Algo defn

$\Rightarrow$ $\exists$ m s.t. w has not proposed to m ——— (*)

Lemma 4

Since S is output

$\Rightarrow$ Algo has terminated $\Rightarrow$ contradicts (*)

condition on the while loop

Pf of Lemma 4

Pf: idea: Proof by contradiction.

(Use Obs 1 + pigeonhole principle)

If n-1 pigeons are assigned to n holes $\Rightarrow \exists \geq 1$ hole with no pigeons

Pf details. For sake of contradiction, assume ∃ a free woman w who has proposed to all the n men.

$\Rightarrow$ all men are engaged.

By Obs 1 + alg. defn $\Rightarrow$ n engaged men but $\leq n-1$ engaged women.

$\Rightarrow$ contradicts pigeonhole principle (if men $\equiv$ holes, women $\equiv$ pigeons)

LEMMA 3: S has no instability.

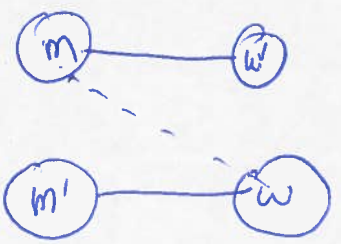
Pf idea: Proof by contradiction (case analysis + Obs 2)

Pf details: For contradiction assume ∃ $(m, w) \notin S$ s.t.

(1) $m > m'$ in L_w

AND

(2) $w > w'$ in L_m



Case 1: w never proposed to m.

Since (m', w) are engaged $\Rightarrow$ w proposed to m' (but not m)

$\Rightarrow$ $m' > m$ in L_w $\Rightarrow$ contradicts (1)!

Obs 2

Case 2: w proposed to m.

Case 2.1: m accepted w's proposal

$\Rightarrow$ (m, w) were engaged.

but later (m, w') get engaged

$\Rightarrow$ (Obs 1) $w' > w$ in L_m $\Rightarrow$ contradicts ②!

Case 2-2: m rejected w_s proposal

$\Rightarrow$ (by Algo dfn) (m, w'') were engaged at that time

$w'' > w$ in L_m

If $w'' = w'$ $\Rightarrow$ contradicts ②!

If not, by Obs 1 $\Rightarrow$ $w' > w''$ in L_m

$\Rightarrow$ $w' > w$ in L_m

$\Rightarrow$ contradicts ②!
