Sep 12 $T_A(N)$ = max # steps taken by A over all inputs of size N.
($T(N)$)

$T(N) = 1.6N^2 + 5N + 60$
or $8N^2 + 5N + 59$ } $\approx N^2$

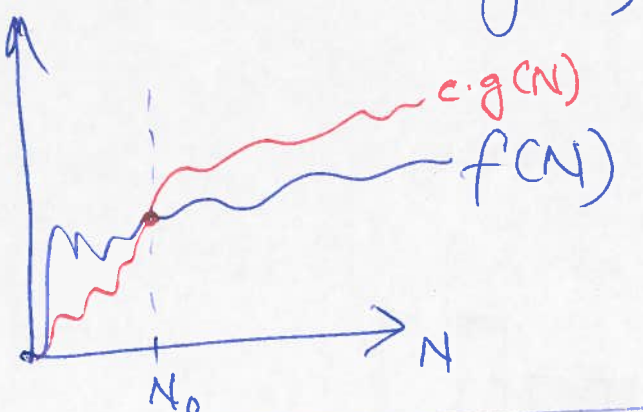
SEARCH $(b; a_1, \ldots, a_n)$
for $i = 1 \ldots n$
If $b = a_i$
Return i
Return -1

$O(n)$ $N = n+1$
$\approx O(N)$

Asymptotic notation

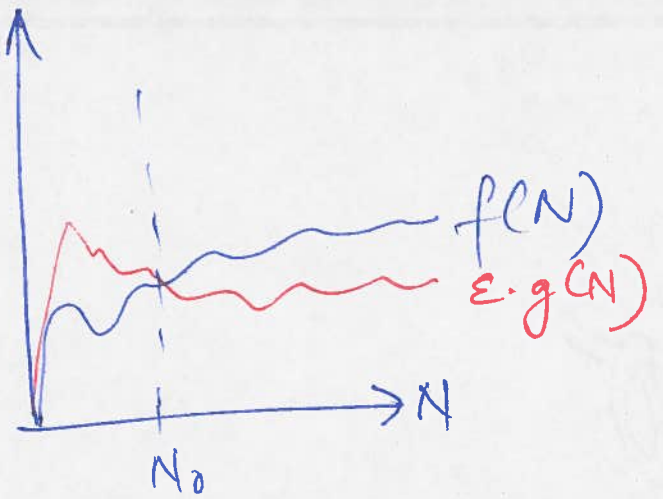
$N \geq 1$ integers

Def: $f(N)$ is $O(g(N))$ if $\exists$ constants $c, N_0 > 0$ (indep. of N) s.t. $\forall N \geq N_0$
$f(N) \leq c \cdot g(N)$



$8N^2 + 5N + 1 \rightarrow O(N^2)$ ✓
$\rightarrow O(N)$ ✗
$\rightarrow O(N^3)$ ✓

Def: $f(N)$ is $\Omega(g(N))$ if $\exists$ constants $\varepsilon, N_0 > 0$
s.t. $\forall N \geq N_0$ $f(N) \geq \varepsilon \cdot g(N)$

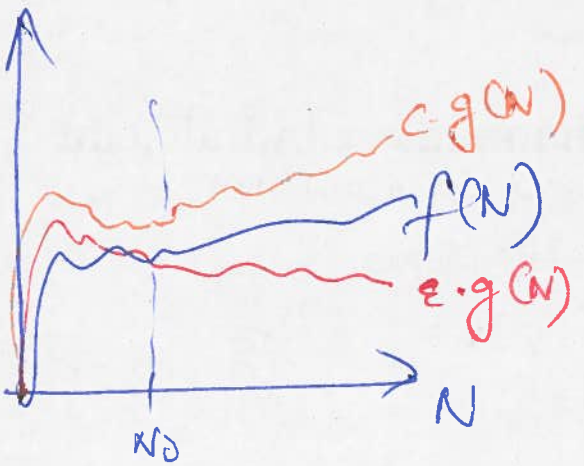


$3N^2 + 5N + 1 \rightarrow \Omega(N^2)$ ✓

$\rightarrow \Omega(N)$ ✓

$\rightarrow \Omega(N^3)$ ✗

Def: $f(N)$ is $\Theta(g(N))$ if $f(N)$ is $O(g(N))$ & $\Omega(g(N))$



$3N^2 + 5N + 1 \rightarrow \Theta(N^2)$ ✓

$\rightarrow \Theta(N)$ ✗

$\rightarrow \Theta(N^3)$ ✗

Common runtimes: $O(1)$, $O(\log N)$, $O(N)$, $O(N \log N)$, $O(N^2)$, $O(N^3)$, $O(2^N)$, $O(N!)$

Property 1: If $f(N)$ is $O(g(N))$ & $g(N)$ is $O(h(N))$

$\Rightarrow f(N)$ is $O(h(N))$

$3N^2$ is $O(N^2)$

$5N$ is $O(N^2)$ $\Leftarrow$ [$5N$ is $O(N)$; N is $O(N^2)$]

Property 2: g is $O(k)$ & f is $O(h)$

$g + f$ is $O(h)$ $3N^2$ is $O(N^2)$ $5N$ is $O(N^2)$

$\Rightarrow 3N^2 + 5N$ is $O(N^2)$

Property 3: f g is $O(h_1)$ & f is $O(h_2)$

$g \cdot f$ is $O(h_1 \cdot h_2)$

N^2 is $O(N^2)$

N is $O(N^2)$

$\Rightarrow N^3$ is $O(N^4)$

All properties also hold for Ω, Θ