

Sep 15, 2014 | $T(N) = \max \# \text{ steps over all inputs of size } N.$

$N = n+1$

SEARCH ($b; a_1, \dots, a_n$)

for $i = 1 \dots n \leftarrow T_0$ (# iteration) $T_0 \leq n; T_1 \leq O(i)$

 if $b = a_i$
 Return i T_1 $T(N) \leq T_0 T_1 + T_2$ $T_2 \leq O(1)$

Return -1. T_2

Want $\max_b \{t_1, \dots, t_m\} \leq U \leftarrow O(n)$
 \uparrow time on the i th input of size n

$\Leftrightarrow \forall i \quad t_i \leq U.$

$T(N) \leq n \cdot O(1) + O(1) \leq O(n) + O(1) \leq O(n).$

PROPOSITION: $T(n)$ is $\Omega(n)$ \downarrow $T(n)$ is $\Theta(n)$

$\max \{t_1, \dots, t_m\} \geq L \geq \Omega(n)$

$\Leftrightarrow \exists i \text{ s.t. } t_i \geq L.$

$n: a_i = i \quad ; \quad b = n+1$

$T(n) \geq T_0 \cdot T_1 \quad (T_1 \geq n \geq \Omega(n))$

$\geq \Omega(n) \cdot \Omega(1) \quad (T_2 \geq 1 \geq \Omega(1))$

$\Rightarrow \Omega(n)$

GS algo implementation

Q1) How do we represent the input

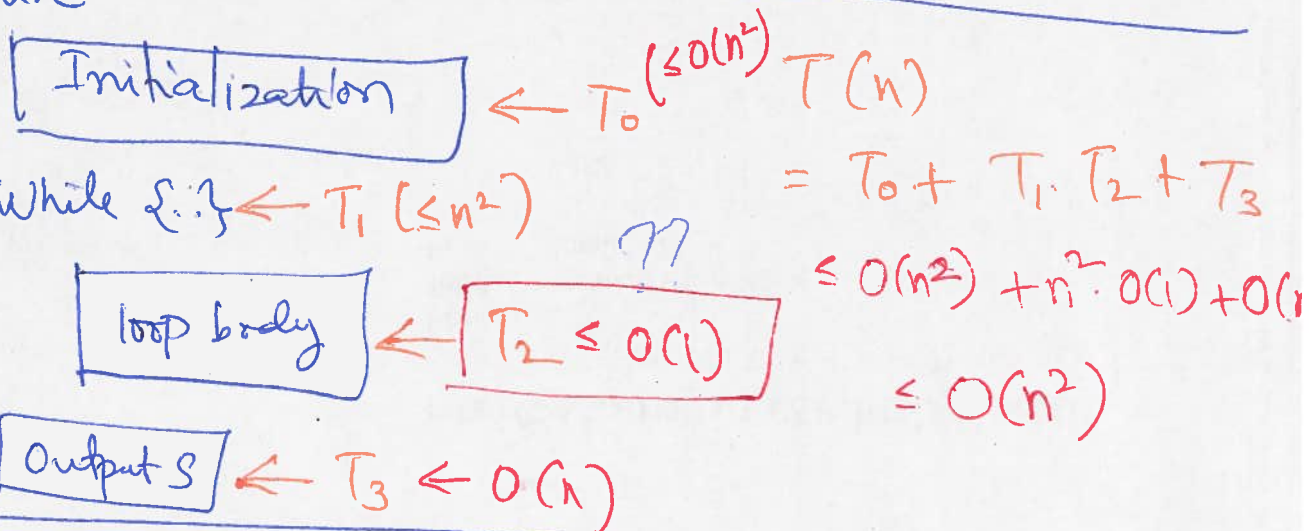
Q2) How do we find a free woman w ?

Q3) How do we find the best ~~up~~ unproposed man m for w ? $O(1)$

Q4) How do we know who m is engaged to?

Q5) How do we decide if $w' > w$ in L_m ?

Algo outline



$M = \{1, \dots, n\}$ $W = \{1, \dots, n\}$
 $[n] \stackrel{\text{def}}{=} \{1, \dots, n\}$

woman $i \rightarrow$ i th woman
 man $i \rightarrow$ i th man

Data structures: Initialization; Query; Update
 Goals: $O(n^2)$; $O(1)$; $O(1)$

Q1) n arrays for men

ManPref $[m][i] \rightarrow$ i th ranked woman for m
 WomanPref $[w][j] \rightarrow$ j th man for w

Initialization: n/a ; Query: Reading an entry $O(1)$; Update: n/a

Q2) Linked list of free women.

Init: Add all n women to list : $O(n)$

Query: Read 'front' : $O(1)$
Delete the front.

Update: Add new free woman at:
front : $O(1)$

Q3) Array of length n

$Next[w] \rightarrow$ best unproposed man's ranking
in $WomanPref[w]$

Best unproposed man $WomanPref[w][Next[w]]$