CS/Mth 237
Final Exam
December 21, 1995
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There are 4 problems, worth equal weight.
Only turn in necessary, labeled, output.

1. Let $x^2 + y^2 = 1$ be a circle of radius 1 with center at the origin. Write a short MATLAB program which generates uniformly distributed random planar points (x, y), where both x and y are in the range of $[0, 1]$. (Note that the matlab command 'rand' can be used to generate a uniformly distributed value in a given range.) Let R be the ratio of the number of points in the positive quadrant that are inside the quarter-circle with respect to the total number of points generated. Write a program to determine an approximation to the value $R = \pi/4$ and show the number of points that your program needs to give such an approximation correctly to 2 decimal points.

2. Consider the linear system

$$\begin{aligned} 1.91x + .956y &= 1 \\ .637x + .333y &= .437 \end{aligned}$$

   Assume these equations are exact to three decimal places.

   (a) Use MATLAB to plot each of these equations as lines in the same window, where the range of the x values is between $x = -4$ and $x = 2$. Place a 'grid' on top of the plot and 'zoom' to an area where the lines intersect. Estimate the intersection point to within 0.25 on the x-axis.

   (b) What is the 'condition' of these equations? [Recall that the condition of a linear system of equations is a measure of how well the system defines its solution; the larger the condition the worse the system. The MATLAB command 'cond(A)' returns the condition number of the matrix A.]

   (c) Compute the 'exact' solution. You may use a calculator, MATLAB, or any method you are comfortable with, but show all your steps, and be sure your answer is accurate up to 5 decimal digits. For example, you might follow the steps in item (d) below, but performing the arithmetic to at least 5 places.

   (d) Use 2-digit floating point arithmetic, (compute the following steps and round at each step to 2 digits) to perform the following:

      i. Solve the first equation for y in terms of x.
      ii. Substitute this expression for y into the second equation.
      iii. Solve the resulting equation for x.
      iv. Using this value for x, find y.
      v. Your answer here will differ from the 'true' solution you found previously. Explain.

3. To see the effect of finite precision arithmetic in a 32 bit calculation, consider the recursion relation

$$\begin{aligned} E_0 &= e - 1 \\ E_n &= e - nE_{n-1} \ . \end{aligned}$$

Write a FORTRAN program that computes $E_0, E_1, \ldots E_{20}$ in

   (a) single (real) precision and

   (b) double precision.

   [In FORTRAN, 'exp(x)' and 'dexp(x)' are the exponential functions e^x, in single and double precision, respectively.] The correct value of

$$E_{10} = 1334961e - 3628800;$$

What is the relative percentage error between this value of E_{10} and *i*) the single precision calculation and *ii*) the double precision calculation that your program produces? [If a is the computed answer and c the correct answer, the relative percent error $= 100\frac{a-c}{c}$.] Hand in hardcopy of your code and output; submit your code.

4. Copy the file ~pitman/final/points.data into your directory. This file contains points (x_i, y_i) in the plane. One "best fit" of a line to these points is the so-called *least squares line*, which is the line that minimizes the sums of the distances from the data points to the line. Compute the least squares fit to a line. Using MATLAB, plot the data points (with stars) and the least squares line.

   The formulas for a least squares fit are as follows:

   The "best-fit" line through N data points (x_i, y_i) is given by the line

$$L(x) = a_1 + a_2 x$$

where

$$a_1 = \frac{\sum_{i=1}^{N} y_i \sum_{i=1}^{N} {x_i}^2 - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i x_i}{N \sum_{i=1}^{N} {x_i}^2 - (\sum_{i=1}^{N} x_i)^2}$$

$$a_2 = \frac{N \sum_{i=1}^{N} y_i x_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N \sum_{i=1}^{N} {x_i}^2 - (\sum_{i=1}^{N} x_i)^2}$$