CS4/531 **Final Exam** Winter, 1997

This exam is *closed book/notes/neighbors/etc.* Answer all questions. *Unsupported work will receive no credit.*

1. (10%) Prove that $\sum_{k=1}^{n} k^{1/6} = \Theta(n^{7/6})$.
2. (10%) Define a *bitonic sequence*.
3. Suppose you are given n points in the plane corresponding to a sampling of a curve taken at n equidistant cardinal points. Further, suppose these points are given in order, left to right, so that the data simply consists of an ordered (by x-coordinate) list of y-coordinates. Determine all *local peaks*, where a local peak y_k is defined as a point such that the sequence $\{y_{k-t}, y_{k-t+1}, \ldots, y_k\}$ is nondecreasing and the sequence $\{y_k, y_{k+1}, \ldots, y_{k+t}\}$ is nonincreasing, for a given *peak width* of t. Give efficient solutions to this problem on the following architectures. Discuss the quality of each of your solutions in terms of time/space/processors.
   a) (10%) RAM
   b) (10%) Mesh
   c) (10%) PRAM
4. Given a set of n line segments, each of which is either horizontal or vertical, give an efficient algorithm to determine whether or not any line segments intersect. Give efficient solutions to this problem on the following architectures. Discuss the quality of each of your solutions in terms of time/space/processors.
   a) (10%) RAM
   b) (10%) Mesh
5. Define a *Reconfigurable Array of size n* to be a linear array augmented by a *reconfigurable bus*. The reconfigurable bus has the characteristic that every processor has the ability to split or connect the bus between itself and either (or both or neither) of its neighbors during *every step of an algorithm*. That is, the bus can be dynamically reconfigured on the fly. Further, on each bus segment, only 1 processor can perform a write and all processors can read the value written (i.e., the bus is CREW within each independent segment).
   a) (10%) Suppose every processor contains either a 1 or a 0. Give an efficient algorithm to determine the LOGICAL-OR of the set of data. That is, determine whether or not there exists at least one "1".
   b) (10%) Suppose every processor contains either a 1 or a 0. Give an efficient algorithm to determine the sum of these values.
   c) (10%) Suppose every processor contains either a 1 or a 0. Give an efficient algorithm to label the connected components. Notice that every component is just a 1-dimensional string of 1's of maximal length.