CS4/531
Spring 1997
Homework #1

(No credit will be given for unsupported answers.)

1. Rank the following by growth rate:
   n, $n^{1/2}$, $\log n$, $\log(\log n)$, $\log^2(n)$, $(1/3)^n$, 4, $(3/2)^n$, $n!$
2. Prove or disprove each of the following.
   a) $f(n) = O(g(n)) \Rightarrow g(n) = O(f(n))$
   b) $f(n) + g(n) = \Theta(\min(f(n), g(n)))$
   c) $f(n) = O((f(n))^2)$
   d) $f(n) = O(g(n)) \Rightarrow g(n) = \Omega(f(n))$
   e) $f(n) + o(f(n)) = \Theta(f(n))$
3. Use O, o, Ω, ω, ***and*** Θ to describe the relationship between the following pairs of functions:
   a) $\log^k n, n^\varepsilon$
   b) n^k, c^n
   c) $2^n, 2^{n/2}$
4. Prove that $17n^{1/6} = O(n^{1/5})$.
5. Prove that $\sum_{k=1}^{n} \frac{1}{k} = \Theta(\log n)$.