

**CS531**  
**Spring 1998**  
**Homework #1**

(No credit will be given for unsupported answers.)

1. Rank the following by growth rate:  
 $n, n^{1/2}, \log n, \log(\log n), \log^2(n), (1/3)^n, 4, (3/2)^n, n!$
2. Prove or disprove each of the following.
  - a)  $f(n) = O(g(n)) \Rightarrow g(n) = O(f(n))$
  - b)  $f(n) + g(n) = \Theta(\min(f(n), g(n)))$
  - c)  $f(n) = O((f(n))^2)$
  - d)  $f(n) = O(g(n)) \Rightarrow g(n) = \Omega(f(n))$
  - e)  $f(n) + o(f(n)) = \Theta(f(n))$
3. Use  $o, \omega, \Omega, \omega, \text{and } \Theta$  to describe the relationship between the following pairs of functions:
  - a)  $\log^k n, n^e$
  - b)  $n^k, c^n$
  - c)  $2^n, 2^{n/2}$
4. Prove that  $17n^{1/6} = O(n^{1/5})$ .
5. Prove that  $\sum_{k=1}^n \frac{1}{k} = \Theta(\log n)$ .