CS531 HW 3 Solution

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1. In the star shaped architecture, there is no way you can communicate with each other without going through PE_0. If each PE only holds one entry, the time to do the semigroup operation is

$$(n-1)+(n-1)+(n-1)$$

The first $n-1$ is for the communication to from PE_i to PE_0, the 2nd $n-1$ is for the computation of semigroup and the last communication is for the communication fro PE_0 to PE_i, which is asympotitacally identical to the serial computer algorithm. If we want to make the architecture useful, the only way I can think of is to put a great amount data $k >> n$ into the architecture.

2. (a) The PEs are grouped into two groups A and B as follows

```
o    o
o    o
o    o
o    o
A    B
```

The semigroup operation can be done in $\lg n$ time.

We can exchange the info and calculate the following fashion

```
0  1          01   01          0123  0167          0-7  0-7
2  3          23   23          2345  0123          0-7  0-7
4  5  --->    45   45   --->   4567  2345   --->   0-7  0-7
6  7          67   67          0167  4567          0-7  0-7
```

The algorithm can be written as the following : the variable mid is the id of myself, cid is the ihe id of the PE which the PE communicates in this phase, n is the number of PEs.

```
pcid = mid
for(i = 0; i < lg n; i++) {
  if(myid \% 2 == 0) cid = (pcid + pow(2, i)) \% n;
  else cid = (pcid - pow(2, i)) \% n;
  exchange info the cid : we have dm (my data) and dr (remote data)
  dm = dm + dr /* Semigroup operation here */
  pcid = cid;
}
```

(b) Assume that after we remove the edges from the PEs, the PEs will be split into two set : set 1 and set 2. If there are x of PEs in the set A belong to set 1, then in set B, there must be $n/2 - x$ of PEs belong to set 1 since the total number of the PEs in set 1 are $n/2$. Similarly, the number of PEs in set A which belong to set 2 will be $n/2 - x$ and x in set B.

```
        -----       -----
          ^           ^
          |           |
set 1     x       n/2 - x set 1
          |           |
          v           v
        -----       -----
          ^           ^
          |           |
set 2   n/2-x         x    set 2
          |           |
          v           v
        -----       -----
        Set A       Set B
```

The the edge need to be cut off in order to make two disjoint set will be

$$f(x) = x^2 + (n/2 - x)^2 = 2x^2 - nx + n^2/4$$

The minumum occurs at the point of $f'(x) = 0$, which is

$$f'(x) = 4x - n = 0, x = n/4$$

If n is divisible by 4, then $x = n/4$, however, if n is not divisible by 4, $x = \lceil n/4 \rceil$ or $\lfloor n/4 \rfloor$
Then replace x back to $f(x)$, we will get the bisection width of $2*(n/4)^2 - n*n/4 + n^2/4 = n^2/16$
The maximum number of the edges in any graph are $n \cdot (n-1) = \Theta(n^2)$ which is asymptotically identical to the bisection width. Therefore the architecture is pretty much connected.